Hedging Long-term Commodity Risk With Dynamic Hedging Strategy

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B.Sc., Nankai University, 2009
M.Sc., The University of British Columbia, 2011

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE

in

The Faculty of Graduate Studies

(Mathematics)

THE UNIVERSITY OF BRITISH COLUMBIA

(Vancouver)

April 2011

Abstract

This essay focuses on the problem of hedging a long-term commitment to deliver a fixed amount of commodity, which often arises when the maturity of actively trading futures contracts on this commodity is limited to a few months. Problem is given and illustrated by real-world examples. Different hedging strategies are introduced. Finally, three stochastic pricing models proposed by Schwartz [17] and Cortazar and Schwartz [7] are discussed in detail and a new way proposed by Lautier and Galli [12] to calibrate the models which takes into account the error associated with the hedge ratios is reviewed.

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Acknowledgements

This essay presents the work under the supervision of Dr. Ivar Ekeland. I would like to express my appreciation for his great guidance, suggestions, patience and encouragement. I would also like to thank my parents and my friends for their love and support.

Chapter 1

The Commodity Hedging Story

The Metallgesellschaft incident of December 1993 has led to interest in the related issues of the extent to which long dated commitments to deliver (or receive) a fix amount of a commodity can be hedged by rolling over a series of short term futures contracts, and how this can be best accomplished. Indeed, such strategies are important for almost every derivative market, especially when long-term transactions may be undertaken, like in interest rate, credit, or commodity markets. In this Chapter, I will try to introduce the hedging problem with a simple case and illustrate the importance of the problem with Metallgesellschaft's debacle.

1.1 Introduction to the Hedging Problem

Consider at a date t, a trader sells forward a barrel of crude oil for delivery at T. He does not have this barrel now and decides to wait until a few days before delivery to buy it on the spot market, thereby avoiding the storage costs associated with the commodity between t and T, but risking a rise in spot prices.

In order to protect himself against fluctuations in the spot price, the trader initiates a dynamic hedging strategy on the futures market. However, the maturity of the position hold on the paper market is shorter than the one of the commitment because of the lack of liquidity on the longer maturities in most derivative markets, or because the available futures contracts do not have a sufficient maturity to cover the position held on the physical market (as was the case for Metallgesellschaft). The gap between the maturities of the physical and paper positions leads naturally to a stack and roll strategy. Indeed, the use of shorter maturities supposes that the hedge portfolio is rebalanced regularly as the futures expiry date approaches, in order to constantly maintain the position on the futures market, where basis risk combined with rollover risk rise.

1.2 Metallgesellschaft's Hedging Debacle

1.2.1 Introduction

Metallgesellschaft Corporation (MG) is the subsidiary of Metallgesellschaft A.G., a German conglomerate with 15 major subsidiaries closely held with over 65% of stock owned by institutional investors including banks. In 1993, MGs trading subsidiary, MG Refining and Marketing (MGRM), established very large energy derivatives (futures and swaps) positions to hedge its price exposure on its forward-supply contracts to deliver gasoline, diesel fuel and heating oil (about 160 million barrels) to its customers over a period of ten years at fixed prices. The counter-parties to forward contracts were retail gasoline suppliers, large manufacturing firms, and some government entities. The central premise of their forward contracts is to supply oil at fixed price to independent retailers who often face severe liquidity crisis and squeezes on margin when oil prices rise. It believed it is possible to arbitrage between the spot oil market and the long-term contract market. This arbitrage required skilled use of the futures markets in oil products, and this was to be MGRMs stock in trade.

1.2.2 The Hedging Situation

MGRM developed several novel contract programs. First, it offered a firm-fixed program under which the customer would agree to a fixed monthly delivery of oil products at a set price. (102 million barrels of oil products were obligated under this program by September 1993). Second, it offered a firm-flexible contracts under which the customers were given extensive rights to set the delivery schedule for up to 20% of its needs in any year, besides the fixed price commitments. (A total of 52 million barrels were contracted under this program). Third, it offered a guaranteed margin contracts under which it agreed to make deliveries at a price that would assure the independent operator a fixed margin relative to the retail price offered by its geographical competitors. The contracts could be extended annually for a defined period and at MGRMs discretion. This means they were not firm obligations. (By September 1993, a total of 54 million barrels were committed under this program.) It is the first two programs involving 154 million barrels of obligations for periods up to ten years that constituted MGRMs designated short position in oil.

Most of the forward contracts were negotiated during the summer of 1993 when energy prices were low and falling and the contracts came with cash-out option if the energy price were to rise above the contractually fixed prices. The fixed delivery prices were set 3 to 5 dollars higher than the spot price when writing the contracts. Under the cash-out provision, the buyer could choose to sell the remainder of its forward obligations back to MGRM for a cash payment of one-half the difference between the prevailing near month futures price and the contractually fixed supply price times the total volume remaining on the contract. MGRM opted for early exercise sell-back options instead of negotiated unwinding. These options take effect when the front-month futures rises above the fixed delivery price in the flow contract. Although customers might wish to exercise these sell-back options, if they expect spot prices in the future to fall, they might well wish to do so even if they regarded a surge in spot prices as permanent. Remember, that they must compare the immediate cash payment with the PV of expected future difference between spot prices and the delivery prices over the remaining life of the contract.

1.2.3 Critics

MGRMs fixed price forward delivery contracts exposed it to the risk of rising energy prices. MGRM hedged this price risk with energy futures contracts of between one to three months to maturity at NYMEX and OTC swaps. The objective of its hedging strategy was to protect the profit margins in its forward delivery contracts by insulating them from increases in energy prices. MGRM would gain substantially from its derivative positions if the energy prices rise. During the later part of 1993, however, energy prices fell sharply ($19 a barrel in June 93 to $15 a barrel in Dec. 93) resulting in unrealized losses and margin calls on derivative positions in excess of $900 million. To complicate the matter, the futures market went into a contango price relationship for almost entire year in 1993 increasing cost each time it rolled its derivatives. The MGs Supervisory Board responded to the situation of mounting margin calls by replacing MGs top management and liquidating MGRMs derivative positions and forward supply contracts which ended MGs involvement in the oil market. It suffered derivative related loss of $1.3 billion by the end of 1993. Only a massive $1.9 billion rescue operation by 150 German and international banks kept MG from going into bankruptcy.

One reason for not buying forward contracts for the same maturity is that market for long-dated oil contracts is small - only about 10 firms made prices in this market. Another reason is that the MGs credit rating was low enough for those firms to be exposed to it for long. (Economist) If energy prices had risen rather than fallen, MGRM would not have had a problem. It would have had unrealized gains on its derivatives position, and positive margin flows from the forward contracts. Although it would have had unrealized losses on its forward contracts, it would not have mattered as it would be offset by the unrealized gains on its derivative positions.

MGRMs hedging strategy included short-dated energy futures contracts and OTC swaps – a stack and roll or rolling stack strategy. Under this s-trategy, MGRM opened a long position in futures staked in the near month contract. Each month MGRM would roll the stack over into the next n-ear month contract, gradually decreasing the size of the position. Under this plan the total long position in the stack would always match the short position remaining due under the supply contracts. It bought long futures positions on the NYMEX (equivalent to 55 m. barrels of gasoline, heating oil and crude oil) and entered into OTC energy swaps (100 to 110 m. barrels) with swap dealers (mostly banks) entitling it to receive payments based upon floating energy prices while making fixed payments. MGRMs total derivatives position was almost equal to its forward commitments, a barrel for barrel hedge or with hedge ratio of one. The short- term nature of the derivatives called for continuous roll forward to maintain the hedge position. This exposed the firm to rollover risk. A stack hedge refers to a futures position being stacked or concentrated in a particular delivery month (or months) rather than being spread over many delivery months. The stack and roll strategy can be profitable when markets are in backwardation, that is, when spot prices are higher than futures prices. But when markets are in contango, that is, when futures prices are higher than spot prices, the strategy will result in losses.

Critics assert that MGRMs strategy exposed it to three significant and related risks: rollover risk, funding risk, and credit risk, because of the maturity mismatch between the hedge and the delivery contracts and other features. It was exposed to rollover risk because of uncertainty about whether it would sustain gains or losses when rolling its derivatives position forward. It is exposed to funding risk because of the marked-to-market conventions that applied to its short-dated derivatives position. It is exposed to credit risk because of its forward delivery counter-parties might default on their long-dated obligations to purchase oil at fixed prices. If the energy prices fell, this risk is expected to increase because of the increase in the difference between contractual prices and prevailing spot prices. To minimize the credit risk, MGRM limited the annual volume supplied under contract to no more than 20% of the customers needs and included in the contracts a cash-out option. It could, however, be a factor in MGRMs ability to raise funds against the collateral of these contracts.

Another feature of MGRMs hedging strategy, which entails mismatched maturity structure, is that it exposed the firm to excessive amount of basis risk-variations in the value of the short-dated futures positions not compensated by equal and opposite variations in the value of the long-dated delivery contracts because of a one-for-one hedge it entailed. One barrel of oil for delivery in one month is simply not equal in PV to one barrel of oil for delivery in ten years and the value of two different dated obligations do not move in lock step. In general, spot prices are more variable than the futures prices. This is a feature that all hedgers must deal with. Hedgers in the futures market are speculators on the basis, trading greater price risk for a lessor basis risk. The basis risk is the difference between the price of the instrument and the price of the underlying asset being hedged.

A rolling stack of short-dated futures initially increases the variance of cash flows. This occurs because movements in the price of oil within the month create losses or gains on the entire stack of contracts. These losses or gains must be settled by the end of the month; while compensating gains or losses on deliveries are realized only gradually over the remaining ten years of the delivery contract. When cash flows matter, the rolling stack may be worse than no hedge at all.

1.2.4 Consequence

MGRM has been losing money on its futures position throughout 1993. The consequences had already been felt within the U.S subsidiary by the end of the summer as the firms credit lines were used up. When the oil price fell yet more precipitously at the end of the year, the company did not have sufficient cash to continue to roll over its stack of oil futures contracts as planned and could not meet a large number of its other obligations until it received an emergency line of credit from its bankers. Losses eventually totaled nearly $1.3 billion. By January the firm was close to declaring bankruptcy and its future was not clear.

MG eventually negotiated a $1.9 billion bailout from its bankers in tandem with a plan to shed assets such as its auto parts manufacturing business, its tin mining operations, its recently acquired heating equipment and others. The price of MG share fell by half between November 1993 and February 1994 as a consequence.

Chapter 2

An Overview of The Hedging Strategy

2.1 Theoretical Models

As a rule, most existing studies of hedging attempt to solve this problem by finding a strategy that minimizes the risk of changes in the spot price, using a certain number of futures contracts with arbitrary constant maturity. Initially, there are several simple hedging models, such as naive-hedge model, price-sensitivity model and minimum-variance model. Enlightening as they may be, these models mainly suffer from basis risk. An even more crucial problem is that the spot prices of many commodities are unobservable, which make these models more unrealistic. In recent year, a large amount of literature has been trying to deal with this problem by modeling the stochastic behaviour of the underlying assets.

Stochastic models of the behavior of commodity prices differ on the role played by the convenience yield[1] and on the number of factors used to describe uncertainty. Early models assumed a constant convenience yield and a one-factor Brownian motion (Brennan and Schwartz [5]). This random walk specification for commodity prices was used until a decade ago, when mean reversion in spot prices began to be included as a response to the evidence that volatility of futures returns declines with maturity. One-factor mean reverting models can be found for example in Laughton and Jacoby [11], Ross [14] and Schwartz [17].

An undesirable implication of one-factor models, however, is that all futures returns are perfectly correlated, a fact that defies empirical evidence. To account for a more realistic stochastic behavior, two-factor models with mean reversion were introduced. Examples are Gibson and Schwartz [8]), Schwartz [17] and Schwartz and Smith [16].

[1] Futures prices are normally lower than the spot price plus the interest rate over the life of the futures contract. This shortfall, which is like an implicit dividend that accrues to the holder of the spot commodity but not to the holder of the futures contract, is what is known as the convenience yield of the commodity (Brennan [2]).

Even though two-factor models behave reasonably well most of the time in the sense that they fit well the cross section of futures prices, for some market conditions they behave poorly. That is, for some days the two-factor model is unable to fit well the cross section of futures prices, suggesting the need for an additional factor.

To further enhance the feasibility of term structure models, three-factor mean models were proposed. Schwartz [17] presents a three-factor model with the third factor calibrated using bond prices. Cortazar and Schwartz [6] also develop a three-factor model using a no-arbitrage approach. Cortazar and Schwartz [7] proposed a three-factor model related to Schwartz [17] with all three factors calibrated using only commodity prices.

2.2 Empirical Literature

The various strategies differ from each other mainly in the assumptions concerning the behaviour of futures prices. So studies of hedging strategies are above all associated with theoretical studies of the term structure, and empirical works have been much rarer.

Brennan and Crew [3] compared the hedging strategy used by Metallgesellschaft on the crude oil market with strategies relying on several term structure models. Studying hedging strategies up to 24 months, they showed that those relying on the term structure models outperform by that of the German firm by far, all the more as the term structure model is able to correctly replicate the price curve empirically observed.

Neuberger [13] compared the performances of hedging strategies based on Schwartzs two factor model and on a new model. In the latter, no assumption is made about the number of variables, the process they follow, or the way risk is priced. The key assumption is that the expected price at which the long dated contract starts trading is a linear function of the price of existing contracts. While theoretically inconsistent with most models of the term structure of commodity prices, this new model still gives good results in practice, when considering hedging strategies up to 36 months. However, it requires balancing the portfolio more frequently than with Schwartzs one.

Routledge et al. [15] took into account the fact that in commodity markets, the basis has an asymmetrical behaviour: it is more volatile in backwardation than in contango. Once again, they computed the hedge ratios based on their term structure model but they did not test dynamic hedging strategies.

Lastly, Veld-Merkoulova and de Roon [18] used a one factor term structure model based on convenience yield to construct hedge strategies that minimize both spot price risk and rollover risk by using futures of two different maturities. They showed that their strategy outperforms the naive hedging strategy. However, they did not compare their results with previous work.

2.3 Lautier and Galli [12]

Lautier and Galli [12] analyses long-term dynamic hedging strategies relying on term structure models of commodity prices and proposes a new way to calibrate the models which takes into account the error associated with the hedge ratios. Different strategies, with maturities up to seven years, are tested on the American crude oil futures market.

Compared to previous works, three of the major contributions of Lautier and Galli [12] are, first to have explicitly calculated the hedge portfolios of the term structure models employed in the study, second, to have found a methodology that leads better optimal hedge ratios than those traditionally employed and third, to have demonstrated its usefulness empirically.

Chapter 3

Underlying Term Structure Models

In this chapter, I will introduce three term structure models of commodity prices. based on Schwartz [17] and Cortazar and Schwartz [7]. Each of these term structure models has specific implications for hedging strategies, and will lead to different hedge ratios, presenting in the next chapter.

3.1 One-factor Model

A futures price is often defined as the expectation of the future spot price, conditional on the available information at date t and under the risk neutral probability. Because the spot price is the most important determinant of the futures price, most one-factor models use it as the single state variable. The model I introduced here is proposed by Schwartz [17], which is currently well-known and extensively used. The model is mean reverting with the spot price being the single state variable.

In the model, the spot price follows mean-reverting process:

$$dS = \kappa(\mu - \ln S)Sdt + \sigma Sdz \tag{3.1}$$

with: κ, $\sigma > 0$

where $-S$ is the spot price,
$-\mu$ is the long-run mean of $\ln S$,
$-\kappa$ is the speed of adjustment of $\ln S$,
$-\sigma$ is the volatility of the spot price,
$-dz$ is the increment of a standard Brownnian motion associated with S.

Defining $X = \ln S$ and applying Ito's Lemma, this implies that the log price can be characterized by an Ornstein-Uhlenbeck Stochastic process:

$$\mathrm{d}X = \kappa(\alpha - X)\mathrm{d}t + \sigma\mathrm{d}z \tag{3.2}$$

with : $\alpha = \mu - \dfrac{\sigma^2}{2\kappa}$.

Under standard assumptions, the dynamics of the Ornstein-Uhlenbeck process under the equivalent martingale measure can be written as:

$$\mathrm{d}X = \kappa(\hat{\alpha} - X)\mathrm{d}t + \sigma \mathrm{d}\hat{z} \tag{3.3}$$

where $\hat{\alpha} = \alpha - \lambda$, λ is the market price of risk (assumed constant) and $\mathrm{d}\hat{z}$ is the increment to the Brownian motion under the equivalent martingale measure.

Under the equivalent martingale measure, the conditional distribution of X at time T is normal with mean and variance:

$$\begin{aligned} E_0[X(T)] &= e^{-\kappa T}X(0) + (1 - e^{-\kappa T})\hat{\alpha} \\ Var_0[X(T)] &= \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa T}) \end{aligned} \tag{3.4}$$

Since $X = \ln S$, the spot price of the commodity at time T follows log-normal distribution under martingale measure with these same parameters. Further assuming that the interest rate is constant, the futures price of the commodity with maturity T is the expected price of the commodity at time T under the equivalent martingale measure, which means:

$$F(S, t, T) = E[S(\tau)] = \exp(E_0[X(\tau)] + \frac{1}{2}Var_0[S(\tau)]) \tag{3.5}$$

Along with Eq. (3.4), we have:

$$F(S, t, T) = \exp\left(\hat{\alpha} + (\ln S - \hat{\alpha})e^{-\kappa\tau} + \frac{\sigma^2}{4}H(\kappa, \tau)\right) \tag{3.6}$$

Or in log form:

$$\ln F(S, t, T) = \hat{\alpha} + (\ln S - \hat{\alpha})e^{-\kappa\tau} + \frac{\sigma^2}{4}H(\kappa, \tau) \tag{3.7}$$

with: $\tau = T - t$, $\hat{\alpha} = \alpha - \lambda$ and $H(x, \tau) = \dfrac{1 - e^{-2x\tau}}{x}$

Two features should be noticed for this one-factor model when the time to maturity of the future contract approaching infinity. The first is that the volatility converges to zero. The second is that the future price converges to a fix value, which is independent of the spot price:

$$F(S, \infty) = \exp\left(\hat{\alpha} + \frac{\sigma^2}{4\kappa}\right)$$

For this model, the term structure is in backwardation when the spot price is above $F(S, \infty)$ and in contango when it's below $F(S, \infty)$. Not allowing for much flexibility in the prices curves is the main limitation of this model.

3.2 Two-factor Model

Convenience yield is the most widely used factor as a second factor in a term structure model. Other factors such as long term price is also considered by some researchers. By introducing a second variable, the model allows for more varied shapes of curves than one factor models and more varied volatility structures. The two-factor model proposed by Schwartz [17] is probably the most famous one, which is an extension of the first one. It's also mean reverting with Convience Yield added as an second state variable.

In the two-factor model, convenience yield C, which can explain the behavior of the futures price, is introduced as a second state variable. The dynamics of these state variables is:

$$\begin{cases} dS &= (\mu - C)Sdt + \sigma_S S dz_S \\ dC &= [\kappa(\alpha - C)]dt + \sigma_C dz_C \end{cases} \tag{3.8}$$

with κ, σ_S, $\sigma_C > 0$

where $-\mu$ is the drift of the spot price,
$-\sigma_S$ is the volatility of the spot price $\ln S$,
$-dz_S$is an increment to a standard Brownian motion associated with S
$-\kappa$ is the speed of adjustment of the convenience yield,
$-\sigma_C$ is the volatility of the convenience yield,
$-dz_C$ is an increment of a standard Brownian motion associated with C.

The increments to standard Brownian motions are correlated, with:

$$E[\mathrm{d}z_S \times \mathrm{d}z_C] = \rho \mathrm{d}t$$

Defining once again $X = \ln S$ and applying Ito's Lemma, the process for the log price can be written as

$$\mathrm{d}X = (\mu - C - \frac{1}{2}\sigma_S^2)\mathrm{d}t + \sigma_S \mathrm{d}z_S \tag{3.9}$$

In this model, the commodity is treated as an asset that pays a stochastic dividend yield C. Thus, the risk adjusted drift of the commodity process will be $r - C$. Since convenience yield risk cannot be hedged, the risk-adjusted convenience yield process will have a market price of risk associated with it. Under the equivalent martingale measure, the stochastic process for the factors can be written as:

$$\mathrm{d}S = (r - C)S\mathrm{d}t + \sigma_S S \mathrm{d}\hat{z_S} \tag{3.10}$$

$$\mathrm{d}C = [\kappa(\alpha - C) - \lambda]\mathrm{d}t + \sigma_C \mathrm{d}\hat{z_C} \tag{3.11}$$

$$\mathrm{d}\hat{z_S}\mathrm{d}\hat{z_C} = \rho \mathrm{d}t \tag{3.12}$$

where now λ is the market price of convenience yield risk, which is again assumed constant. Let $\tau = T - t$ and assuming that the future price $F(S, C, t, T)$ is twice continually differentiable function of S and C. By I-to's Lemma, we have:

$$\begin{aligned} \mathrm{d}F =& F_S \mathrm{d}S + F_C \mathrm{d}C - F_\tau \mathrm{d}t + \frac{1}{2} B_{SS} (\mathrm{d}S)^2 \\ &+ \frac{1}{2} B_{CC} (\mathrm{d}C)^2 + B_{SC} \mathrm{d}S \mathrm{d}C \\ \mathrm{d}F =& [-F_\tau - \frac{1}{2} F_{SS} \sigma_S^2 \mathrm{d}S^2 + F_{SC} S \rho \sigma_S \sigma_C + \frac{1}{2} F_{CC} \sigma_C^2 + F_S \mu S \\ &+ B_C(\kappa(\alpha - \sigma))] \mathrm{d}t + \sigma_S S F_S \mathrm{d}Z_S + \sigma_C F_C \mathrm{d}Z_C \end{aligned}$$

Abstracting from interest rate uncertainty and invoking the standard perfect market assumptions, it can be shown that in the absence of arbitrage the price of the future must satisfy the following partial differential equation (Brennan and Schwartz [4]):

$$\begin{aligned} \frac{1}{2} F_{SS} S^2 \sigma_S^2 + \frac{1}{2} F_{CC} \sigma_C^2 + F_{SC} S C \sigma_S \sigma_C + F_S S (r - C) \\ + F_C (\kappa(\alpha - C) - \lambda \sigma_C) - F_\tau = 0 \end{aligned} \tag{3.13}$$

Subject to the initial condition:

$$F(S, C, t, t) = 0$$

jamsshidian and Fein [10] and Bjerksund [1] have shown that the solution to (3.13) is:

$$F(S, C, t, T) = S(t) \times \exp\left[-C(t) \frac{1 - e^{-\kappa\tau}}{\kappa} + B(\tau)\right] \tag{3.14}$$

Or in log form:

$$\ln F(S, C, t, T) = \ln S(t) - C(t) \frac{1 - e^{-\kappa\tau}}{\kappa} + B(\tau) \tag{3.15}$$

With:

$$B(\tau) = (r - \hat{\alpha} + \frac{\sigma_C^2}{2\kappa^2} - \frac{\sigma_S \sigma_C \rho}{\kappa}) \times \tau + (\frac{\sigma_C^2}{5})(\frac{1 - e^{-2\kappa\tau}}{\kappa^3}) + (\hat{\alpha}\kappa + \sigma_S \sigma_C \rho - \frac{\sigma_C^2}{\kappa}) \times \frac{1 - e^{-\kappa\tau}}{\kappa^2}$$

$$\hat{\alpha} = \alpha - (\lambda / \kappa)$$

With two-factor model, we can obtain sunken curve, humped curve or flat curve. Furthermore, when the contract reaches its expiry date, the future price's volatility converges towards the spot price volatility. Conversely, when the maturity approaches infinity, the volatility of the futures price tends towards a fixed value. The price to pay for increased flexibility is increased complexity, for the reason that the model has six parameters compared with two for the one-factor model.

3.3 Three-factor Model

In 2003, Cortazar and Schwartz [7] proposed a three-factor model related to Schwartz [17]. In this model, the authors consider as a third risk factor the long term spot price return, allowing it to be stochastic and to return to a long term average. The two other stochastic variables are the spot price and the convenience yield. The latter models short-term variations in prices due to changes in inventory, whereas the long term return is due to changes in technologies, inflation, and demand pattern. The dynamic of these state variables is the following:

$$\begin{aligned} \mathrm{d}S &= (v-y)S\mathrm{d}t + \sigma_1 S\mathrm{d}z_1 \\ \mathrm{d}y &= -\kappa y\mathrm{d}t + \sigma_2\mathrm{d}z_2 \\ \mathrm{d}v &= a(\bar{v}-v)\mathrm{d}t + \sigma_3\mathrm{d}z_3 \end{aligned} \tag{3.16}$$

with:

$$\begin{aligned} \mathrm{d}z_1\mathrm{d}z_2 &= \rho_{12}\mathrm{d}t \\ \mathrm{d}z_1\mathrm{d}z_3 &= \rho_{13}\mathrm{d}t \\ \mathrm{d}z_2\mathrm{d}z_3 &= \rho_{23}\mathrm{d}t \end{aligned} \tag{3.17}$$

where $-S$ is the spot price,
$-y$ is the demeaned convenience yield, with$y = C - \alpha$, where α is the long run mean of the convenience yield C
$-v$ is the expected long-term spot price return
$-\kappa$ is the speed of adjustment of the demeaned convenience yield
$-a$ is the the speed of adjustment of v
$-\bar{v}$ is the long-run mean of the expected long term spot price return
$-\sigma_i$ is the volatility of the variable i
$-\rho_{ij}$ is the correlation between the variables i and j
$-\mathrm{d}z_i$ is the increment of a standard Brownian motion associated with the variable i

Under a similar standard argument (Cortazar and Schwartz [7]) as that of the two factor model, it can be shown that the price of the future is:

$$F(s,y,v,t,T) = S(t) \times \exp\left(-y(t)H(\kappa,\tau) + v(t)H(a,\tau) + \phi(\tau)\right) \tag{3.18}$$

with:

$$
\begin{aligned}
H(x,\tau) &= \frac{1-e^{-x\tau}}{x} \\
\phi(\tau) =&\mu\tau + \frac{1}{2}\sigma_3^2[\frac{\tau - H(a,\tau)}{a^2} - \frac{H(a,\tau)^2}{2a}] + \frac{1}{2}\sigma_2^2[\frac{\tau - H(\kappa,\tau)}{\kappa^2} - \frac{H(\kappa,\tau)^2}{2\kappa}] \\
&+ \frac{\sigma_1\sigma_3\rho_{13}}{a}(\tau - H(a,\tau)) - \frac{\sigma_1\sigma_2\rho_{12}}{\kappa}(\tau - H(\kappa,\tau)) \\
&- \frac{\sigma_2\sigma_3\rho_{23}}{a+\kappa}[\tau(\frac{1}{a}+\frac{1}{\kappa}) - \frac{1}{\kappa}H(\kappa,\tau) - \frac{1}{a}H(a,\tau) - H(\kappa,\tau)H(a,\tau)] \\
\mu &= \bar{v} - (\lambda_1 + \lambda_2 + \lambda_3)
\end{aligned}
$$

This volatility term structure decreases with maturity and converges to a positive constant. This is consistent with a mean reverting non stationary process.

Chapter 4

Estimating Model Parameters And Spot Price With Kalman Filter

One of the difficulties in implementing these commodity price models is that the parameters and the state variables of the models are unobservable in many circumstances. For some commodities (such as crude oil), the spot price is hard to obtain, which is usually estimated by the futures price with closest maturity. The convenience yield is obviously unobservable and even more complex to estimate. So is the long term spot price return. The state space form is the appropriate procedure to deal with situations in which the state variables are not oberservable, but are known to be generated by a Markov process. Once a model has been cast in the state space form, the Kalman Filter may be applied to estimate both the parameters of the model and the time series of the unobservable state variables. In this chapter, I will make the one-factor model and two-factor model as examples to illustrate the implementation of the whole methodology.

4.1 Generating State Space Form

The general state space form applies to a multivariate time series of observable variables, meaning futures prices for different maturities in this case, related to an unobservable vector of state variables, in this case the spot price alone or the spot price and the convenience yield, via a measurement equation. Measurement equations can be derived from Eq. (3.7) and (3.15) for the one and two-factor models, respectively. By adding serially and cross-sectionally uncorrelated disturbances with mean zero to take into account bid-ask spreads, price limits, nonsimultaneity of the observations, errors in the data, etc. This simple structure for the measurement errors is imposed so that the serial correlation and cross correlation in the log prices in attributed to the variation of the unobservable state variables. The unobservable state variables are generated via transition equation, which is a discrete time version of the stochastic process for the state variables: Eq. (3.2) for one-factor model and Eq. (3.8) and (3.9) for two-factor model.

4.1.1 State Space form for One-factor Model

From Eq. (3.7) the measurement equation can be written as:

$$y_t = d_t + Z_t X_t + \epsilon_t, \qquad t = 1, \ldots, NT \tag{4.1}$$

where

$$y_t = [\ln F(\tau_i)], \qquad i = 1, \ldots, N, \qquad N \times 1 \text{ vector of variables}$$

$$d_t = [(1 - e^{-\kappa\tau_i})\hat{\alpha} + \frac{\sigma^2}{4\kappa}(1 - e^{-2\kappa\tau_i})], \qquad i = 1, \ldots, N, \qquad N \times 1 \text{ vector}$$

$$Z_t = [e^{\kappa\tau_i}], \qquad i = 1, \ldots, N, \qquad N \times 1 \text{ vector}$$

$$\epsilon_t, \quad N \times 1 \text{ vector of serially uncorrelated disturbances with}$$

$$E(\epsilon_t) = 0, \qquad Var(\epsilon_t) = H$$

and from Eq. (3.2) the transition equation can be written as: [2]

$$X_t = c_t + Q_t X_{t-1} + \eta_t, \qquad t = 1, \ldots, NT$$

where

$$c_t = \kappa\alpha\Delta t \qquad Q_t = 1 - \kappa\Delta t$$

$$\eta, \quad \text{serially uncorrelated disturbances with}$$

$$E(\eta_t) = 0, \qquad Var(\eta_t) = \sigma^2\Delta t$$

[2] The exact transition equation is:

$$X_t = \alpha(1 - e^{-\kappa\Delta t}) + e^{-\kappa\Delta t}X_{t-1} + \eta_t$$

Use weekly data the linear approximation gives the identical parameters estimates up to the fourth significant figure and has been used in all the estimations.

4.1.2 State Space Form for Two-factor Model

From Eq. (3.15) the measurement equation can be written as:

$$y_t = d_t + Z_t[X_t, C_t]' + \epsilon_t, \qquad t = 1, \ldots, NT$$

where

$$y_t = [\ln F(\tau_i)], \qquad i = 1, \ldots, N, \qquad N \times 1 \text{ vector of variables}$$

$$d_t = [B(\tau_i)], \qquad i = 1, \ldots, N, \qquad N \times 1 \text{ vector}$$

$$Z_t = [1, -\frac{1 - e^{-\kappa\tau_i}}{\kappa}], \qquad i = 1, \ldots, N, \quad N \times 2 \text{ matrix}$$

$$\epsilon_t, \quad N \times 1 \text{ vector of serially uncorrelated disturbances with}$$

$$E(\epsilon_t) = 0, \qquad Var(\epsilon_t) = H$$

and from Eq. (3.8) and (3.9) the transition equation can be written as:

$$[X_t, C_t]' = c_t + Q_t[X_{t-1}, C_{t-1}]' + \eta_t, \qquad t = 1, \ldots, NT$$

where

$$c_t = [(\mu - \frac{1}{2}\sigma_S^2)\Delta t, \kappa\alpha\Delta t]', \qquad 2 \times 1 \text{ vector}$$

$$Q_t = \begin{pmatrix} 1 & \Delta t \\ 0 & 1 - \kappa\Delta t \end{pmatrix}$$

$$\eta, \quad \text{serially uncorrelated disturbances with}$$

$$E(\eta_t) = 0, \qquad Var(\eta_t) = \begin{pmatrix} \sigma^2\Delta t & \rho\sigma_S\sigma_C\Delta t \\ \rho\sigma_S\sigma_C\Delta t & \sigma^2\Delta t \end{pmatrix}$$

4.2 Maximum Likelihood Method and Kalman Filter

Schwartz [17] proposed an variable estimation methodology by using Kalman filter along with conditional maximum likelihood method for this problem.

The Kalman filter is a recursive procedure for computing the optimal estimator of the state vector at time t, based on the information available at time t, and it enables the estimate of the state vector to be continuously updated as new information becomes available. When the disturbances and the initial state vector are normally distributed the Kalman filter enables the likelihood function to be calculated, which allow unknown parameters of the model and provides the basis for statistical testing and model specification.

If all the paramers are known, it becomes a standard Kalman filter problem, as described below. Let α_t denote unobservable state variables, y_t denote observable variables.

$$y_t = Z_t\alpha_t + d_t + \epsilon_t \qquad \text{measurement equation}$$

$$\alpha_t = Q_t\alpha_{t-1} + c_t + \eta_t \qquad \text{transition equation}$$

where $\epsilon_t \sim$N(0,H), $\eta_t \sim$ N(0, R) and Z_t, d_t, Q_t, c_t, H and R are all known.

Let a_t be the optimal estimator of α_t based on the observations up to and including y_t. Let P_t denote the covariance matrix of the estimation error, i.e.

$$P_t = E[(\alpha_t - a_t)(\alpha_t - a_t)^{'}]$$

Then, by the theory of Kalman filter, we have the time updating procedure:

$$\begin{cases} a_{t|t-1} = Q_t a_{t-1} + c_t \\ P_{t|t-1} = Q_t P_{t-1} Q_t^{'} + R_t \end{cases} \tag{4.2}$$

and the measurement updating procedure:

$$\begin{cases} P_t = P_{t|t-1} - P_{t|t-1} Z_t^{'} F_t^{-1} Z_t P_{t|t-1} \\ a_t = a_{t|t-1} + P_{t|t-1} Z_t^{'} F_t^{-1} (y_t - Z_t a_{t|t-1} - d_t) \\ F_t = Z_t P_{t|t-1} Z_t^{'} + H_t \end{cases} \tag{4.3}$$

With an specified initial value $\alpha_0 \sim N(a_0, P_0)$, we simply do the procedures recursively to get an optimal estimation of the unobservable state variables.

However, in the context, parameters are unknown, which need to be estimated as well. The principal characteristic of a time series model is that the observations are not independent. Hence classical theory of maximum likelihood is not applicable. Instead, Schwartz [17] suggests to use the definition of conditional probability density function to write the joint density function as[3]

$$L(y; \Phi) = \prod_{t=1}^{T} p(y_t|Y_{t-1}) \tag{4.4}$$

where $p(y_t|Y_{t-1})$ denotes the distribution of y_t conditional on the information at $t-1$, that is $Y_{t-1} = \{y_{t-1}, y_{t-2}, \ldots, y_1\}$.

[3] As Schwartz [17] stated, this method is given by Harvey [9] in Chapter 3.

Conditional on T_{t-1}, α_t is normally distributed with a mean of $a_{t|t-1}$ and a covariance matrix of $P_{t|t-1}$. If the measurement equation is written as

$$y_t = Z_t a_{t|t-1} + Z_t(\alpha_t - a_{t|t-1}) + d_t + \epsilon_t \tag{4.5}$$

it can be seen immediately that the conditional distribution of y_t is normal with mean

$$E_{t-1}(y_t) = y_{t|t-1} = Z_t a_{t|t-1} + d_t$$

and a covariance matrix F_t, given by Eq. (4.3). For a Gaussian model, therefore, the likelihood function of (4.4) can be written as

$$\log L = -\frac{NT}{2}\log 2\pi - \frac{1}{2}\sum_{t=1}^{T}\log|F_t| - \frac{1}{2}\sum_{t=1}^{T} v_t{}' F_t^{-1} v_t \tag{4.6}$$

where

$$v_t = y_t - y_{t|t-1}$$

Thus, with initialized distribution[4] of α_0, we can iteratively do maximum likelihood estimation and Kalman filter as below:

1. With information up to time t, we estimate parameters by Eq. (4.6);
2. With observations at time t+1 and the parameters estimated in the last step, we do Kalman filter to estimate the unobservable state variables at time t+1;
3. With the new information we have of time t+1, we go back to the first step to get an more precise estimation of the parameters.

Convergence might be a problem and there is no proof in Harvey [9]. However, following this methodology, Schwartz [17] showed that all the parameters in the term structure models converge to stable constant[5], suggesting the convergence is not a problem is this case.

[4]Because the sample size is large enough, the specification of initial conditions is unimportant here. Also Harvey [9] gives some other methods to initialize unknown state variables. See Chapter 3 of Harvey [9].

[5]Empirical results are shown in page 937 of Schwartz [17].

Chapter 5

Dynamic Hedging Strategy

5.1 Hedging Ratio

As Lautier and Galli [12] say, to properly hedge the forward commitment, the sensitivity of the present value of the commitment with respect to each of the underlying factors must equal that of the portfolio of futures contracts used to hedge the commitment with respect to the same factors. Since the three models assume constant interest rates, and therefore that futures prices are equal to forward prices, the present value of the forward commitment per unit of the commodity is obtained by discounting the future (forward) price.

The one factor model requires only one position in the futures market:

$$w_1 F_s(S, \tau_1) = e^{-r\tau} F_s(S, \tau) \tag{5.1}$$

The solution of this equation is:

$$w_1 = e^{-r\tau} \frac{F_s(S, \tau)}{F_s(S, \tau_1)} \tag{5.2}$$

The two factor model requires two positions for delivery at time t_1 and t_2:

$$\begin{cases} w_1 F_s(S, C, \tau_1) + w_2 F_s(S, C, \tau_2) = e^{-r\tau} F_s(S, C, \tau) \\ w_1 F_c(S, C, \tau_1) + w_2 F_c(S, C, \tau_2) = e^{-r\tau} F_c(S, C, \tau) \end{cases} \tag{5.3}$$

The solution of the system is:

$$w = e^{-r\tau} Y(\tau) \begin{bmatrix} \frac{H(\kappa,\tau_2) - H(\kappa,\tau)}{(H(\kappa,\tau_2) - H(\kappa,\tau_1))Y(\tau_1)} \\ \frac{H(\kappa,\tau) - H(\kappa,\tau_1)}{(H(\kappa,\tau_2) - H(\kappa,\tau_1))Y(\tau_2)} \end{bmatrix} \tag{5.4}$$

where: $Y(\tau_i) = F_s(S, C, \tau_i)$

The three factor model requires two positions for delivery at time t_1, t_2 and t_3, representing the spot price, the convenience yield, and the long term price, respectively:

$$
\begin{aligned}
w_1 F_S(S,y,v,\tau_1) + w_2 F_S(S,y,v,\tau_2) + w_3 F_S(S,y,v,\tau_3) &= e^{-r\tau} F_S(S,y,v,\tau)\\
w_1 F_y(S,y,v,\tau_1) + w_2 F_y(S,y,v,\tau_2) + w_3 F_y(S,y,v,\tau_3) &= e^{-r\tau} F_y(S,y,v,\tau)\\
w_1 F_v(S,y,v,\tau_1) + w_2 F_v(S,y,v,\tau_2) + w_3 F_v(S,y,v,\tau_3) &= e^{-r\tau} F_v(S,y,v,\tau)
\end{aligned}
\tag{5.5}
$$

The solution of this system is:

$$
w = e^{-r\tau} Y(\tau) \begin{bmatrix} \frac{w_1}{Y(\tau_1)} \\ \frac{W_2}{Y(\tau_2)} \\ \frac{W_3}{Y(\tau_3)} \end{bmatrix} \tag{5.6}
$$

where:

$$
w = \begin{bmatrix}
\frac{H(\kappa,\tau_2)H(a,\tau_3) - H(\kappa,\tau_3)H(a,\tau_2) + H(\kappa,\tau)(H(a,\tau_2) - H(a,\tau_3)) + H(a,\tau)(H(\kappa,\tau_3) - H(\kappa,\tau_2))}{D} \\
\frac{H(\kappa,\tau_1)H(a,\tau_3) - H(\kappa,\tau_3)H(a,\tau_1) + H(\kappa,\tau)(H(a,\tau_1) - H(a,\tau_3)) + H(a,\tau)(H(\kappa,\tau_3) - H(\kappa,\tau_1))}{D} \\
\frac{H(\kappa,\tau_2)H(a,\tau_1) - H(\kappa,\tau_1)H(a,\tau_2) + H(\kappa,\tau)(H(a,\tau_2) - H(a,\tau_1)) + H(a,\tau)(H(\kappa,\tau_1) - H(\kappa,\tau_2))}{D}
\end{bmatrix}
$$

$$
D = (H(\kappa,\tau_3) - H(\kappa,\tau_2))H(a,\tau_1) + (H(\kappa,\tau_1) - H(\kappa,\tau_3))H(a,\tau_2) + (H(\kappa,\tau_2) - H(\kappa,\tau_1))H(a,\tau_3)
$$

5.2 Delta hedge of the forward commitment

Hedging period is decomposed into several subperiods, and new hedge ratios are computed each time the position is rolled on the futures market. So an intermediate hedging error corresponding to each subperiod is computed.

For the two-factor model, if we denote F as the futures price, L as the present value of the forward commitment, τ as the maturity and r as the interest rate observed at t for the maturity τ, the intermediate error $e_{t,\tau}$ between (t-1) and t can be written:

$$
e_{t,\tau} = w_1 \Delta F_1 + w_2 \Delta F_2 - \Delta L_{t,\tau} \tag{5.7}
$$

where:

$$
\begin{aligned}
\Delta L_{t,\tau} &= e^{-r_{t,\tau}\tau} F(t,\tau) - e^{-r_{t-1,\tau+1}(\tau+1)} F(t-1,\tau+1)\\
\Delta F_i &= F(t,\tau_i - 1) - F(t-1,\tau_i)
\end{aligned}
$$

If the hedge was perfect, the error would be equal to zero. A positive error means that the futures positions overestimate the variations in the value of the forward commitment: there is an overhedge. On the contrary, a negative error implies an underestimation of the fluctuations in the value of the commitment.

5.3 Estimation procedure

We know that there are two sources of error in this hedging strategy, the estimation error associated with the term structure model and the intermediate error associated with the hedging ratio.

5.3.1 The errors associated with futures prices and hedge ratios

In order to adjust the sensitivity of the present value of the commitment to that of the hedge portfolio, the models estimations rely on the derivatives themselves instead of the prices, as is usually done. In order to take into account the errors associated with hedge ratios as well as prices' errors, and to control all the dimensions of the first derivatives used to determine the hedge strategy, the weighted error E_t that should be minimized is:

$$\|E_t\|^2 = \beta\|v_t\|^2 + (1-\beta)\|e_t\|^2 \tag{5.8}$$

where $-v_t$ is the difference between the prices estimated with the model and the observed futures prices,
$-e_t$ is the intermediate error of the hedging strategy,
$-\beta \in [0, 1]$ is the weight associated with the price error,
$-1-\beta$ is weight associated with hedge ratios.

When the purpose of the estimation procedure is to determine which model is the best suited to replicate the price curve, the relevant criterion for the estimation is the innovation v_t. The errors associated with the hedge ratios are ignored, β is set to 1, and E_t reflects only prices errors. Conversely, when the objective is to construct optimal dynamic hedging strategies, the prices innovations are not taken into account and β is set to zero. If we were interested in hedging and pricing, we would have to adjust empirically the parameter β.

5.3.2 A focus on the errors associated with hedge ratios

The hedge ratios wi are set to satisfy equations (5.2), (5.4) and (5.6). However, as there is a discrepancy between the model and reality, if we replace the derivatives computed in the model by the actual ones, that is to say, the ones computed from futures prices, there will be an error $\tilde{e}_t$ which changes with the model. For example, for the one-factor model, $\tilde{e}_t$ is:

$$\tilde{e}_t = w_1 F_S^*(S, \tau_1) - e^{-r\tau} F_S^*(S, \tau)$$

The star indicates the use of the true derivatives.

To minimise the error of the hedging strategy, we cannot use the derivatives computed with the model because the error would by definition be zero, which means we would have to use the unkown true derivatives of the futures. At this point we use the fact that in all three term structure models considered in this study, the derivatives are proportional to the futures prices. Thus, we could obtain a new error involving the futures values themselves by multiplying the first equation in each model by $S(t)$. For instance, in the one-factor model, this new error term is:

$$e_t = e^{\kappa\tau_1} w_1 F(\tau_1) - e^{(\kappa - r)\tau} F(\tau)$$

For the other two models we do not have to change the second and third terms, because the derivatives are already expressed as a constant multiplied by the futures value. So, for the two factor model, the term to minimize is:

$$\tilde{e_t} = \begin{bmatrix} \tilde{e_t}^1 \\ \tilde{e_t}^2 \end{bmatrix} = \begin{bmatrix} w_1 F_S^*(S,C,\tau_1) + w_2 F_S^*(S,C,\tau_2) - e^{-r\tau} F_S^*(S,C,\tau) \\ w_1 F_C^*(S,C,\tau_1) + w_2 F_C^*(S,C,\tau_2) - e^{-r\tau} F_C^*(S,C,\tau) \end{bmatrix}$$

Since it's possible to write:

$$F_C(S,C,\tau_i) = -H(\kappa,\tau_i) \times F(\tau_i)$$

Thus e_t can be written as

$$\tilde{e_t} = \begin{bmatrix} e_t^1 \\ e_t^2 \end{bmatrix} = \begin{bmatrix} w_1 F(\tau_1) + w_2 F(\tau_2) - e^{-r\tau} F(\tau) \\ w_1 H(\kappa,\tau_1) F(\tau_1) + w_2 H(\kappa,\tau_2) F(\tau_2) - e^{-r\tau} H(\kappa,\tau) F(\tau) \end{bmatrix}$$

Similarly, for the three factor model, e_t is:

$$e_t = \begin{bmatrix} e_t^1 \\ e_t^2 \\ e_t^3 \end{bmatrix} = \begin{bmatrix} w_1 F(\tau_1) + w_2 F(\tau_2) + w_3 F(\tau_3) - e^{-r\tau} F(\tau) \\ w_1 H(\kappa,\tau_1) F(\tau_1) + w_2 H(\kappa,\tau_2) F(\tau_2) + w_3 H(\kappa,\tau_3) F(\tau_3) - e^{-r\tau} H(\kappa,\tau) F(\tau) \\ w_1 H(a,\tau_1) F(\tau_1) + w_2 H(a,\tau_2) F(\tau_2) + w_3 H(a,\tau_3) F(\tau_3) - e^{-r\tau} H(a,\tau) F(\tau) \end{bmatrix}$$

These errors terms are directly related to our objective to hedge a long-term commitment with short-term futures contracts.

5.4 Performances of The Hedging Strategies

The two authors do empirical work by using crude oil data. The crude oil data are daily settlement prices for the light, sweet, crude oil contract negotiated on the New York Mercantile Exchange (Nymex), from 17 March

1997 to 21 June 2005. Monthly observations on crude oil futures prices were used to estimate the optimal parameters associated with each term structure. The length of these windows corresponds to the maturity of the commitment on the physical market. For the latter, they chose six different delivery dates: 7, 6, 5, 4, 3, and 2 years. The fourth price used for the calibration has a maturity that is identical to the one of the physical commitment. Thus, they took into account first, that the parameters change with time, second that they vary with maturity (Schwartz [17]). Moreover, because the nearest futures price contains a lot of information but may also contain a lot of noise, they always retained the second month maturity for the estimations. Moreover, for the parameter estimations, and for the measure of the performances, four series of futures prices are retained and they use rolling windows. The length of these windows corresponds to the maturity of the commitment on the physical market.

Empirical test showed that taking account of the errors associated with hedge ratios when constructing the portfolios improves the performances of the hedging strategies, while still allowing us to hedge with short-term instruments. It also reduces the mean and standard deviation of the errors, especially for long-term commitments; it drastically decreases the extreme values of the errors; the tendency to under hedge the forward commitment is lessened; fewer futures contracts are required in the hedge portfolio, which would otherwise be a cause for concern with the three-factor model. This is all the more important since the three-factor model is the best one.

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