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Statement of Research interests

I am working as a postdoctoral fellow at the University of British Columbia and my research focuses on mathematical modeling and numerical analysis. One area of interest is in mathematical and numerical modeling of the propagation of hydraulic fractures in an elastic material. We have successfully modeled the change in behavior near the crack tip to give a uniform asymptotic solution across this region for a particular geometry and we are currently extending this work to other geometries and modeling other effects. This work is supervised by Professors Rachel Kuske and Anthony Peirce from the Department of Mathematics. I am also working on with Professor Uri Ascher from the Department of Computer Science on numerical methods for solving shallow water wave equations which exhibit a multi-symplectic structure, namely the Boussinesq equations. Other research includes a continuation of my PhD work with Professors Alastair Spence and Bill Morton which is concerned with the analysis and numerical approximation of systems of differential equations that describe the transport of chemicals in groundwater flow.

Hydraulic fracturing problem

A particular class of fractures in the Earth develops as a result of internal pressurization by a viscous fluid. These fractures are either natural such as volcanic dikes driven by magma from beneath the Earth's crust, or man-made hydraulic fractures created by injecting a viscous fluid from a bore-hole in order to increase production from oil or gas reservoirs. The questions that arise in connection with these problems are simple, yet fundamental: How is the fracture evolving in shape and size? How is the fracturing pressure varying with time? How does the process depend on properties of the rock? These questions are still open due to the formidable challenge of constructing solutions for fluid-driven fractures using either analytical or numerical techniques.

The difficulty of solving this problem originates from the non-linearity of the equation governing the flow of fluid in the fracture, the non-local character of the elastic response of the fracture, and the time-dependence of the equation governing the exchange of fluid between the fracture and the rock.

At present we are involved in the development and application of multiple scale and singular perturbation analysis to the system of governing non-linear integro-differential equations. In addition to variations on the different spatial and temporal scales, there are other scaling relationships between important physical parameters. We have developed a unifying scaling framework which allows analysis of simultaneous effects, namely viscosity, toughness and leak-off, and have successfully modeled the change in behavior near the crack tip to give a uniform asymptotic solution across the near-tip region. This gives us a better understanding of the influence and interaction of the many important factors in fluid-driven fracture, and will enable us to develop more accurate numerical codes of realistic problems than those in existence. Our research to date has focused on plane strain hydraulic fractures on long timescales and we are now applying this method to related models with additional timescales, fluid lag (whereby regions devoid of fluid develop close to the fracture tip), or different geometries such penny-shaped or elliptical fractures.

We have currently submitted two papers for publication which deal with the permeable and impermeable cases separately: the former to the Journal of Applied Mechanics and the latter to SIAM Journal on Applied Mathematics. Pre-prints of both these papers are available at www.iam.ubc.ca/∼sarah.

Numerical modeling of the Boussinesq equations

The Boussinesq equations describe two way propagation of small-amplitude, long wavelength gravity waves on the surface of water in a canal. We are interested in exploiting the multi-symplectic structure by considering discretization schemes that also adhere to such principles: namely conservative, compact schemes that are symmetric in time and space. Standard methods use explicit predictor-corrector methods which do not satisfy this structure. We are currently applying the multi-symplectic box scheme to these systems, which show promise in improving on previous work by allowing us to take larger mesh spacing but still retaining accuracy and stability.

Numerical modeling reactive flow problems

The work based on my PhD thesis showed that key properties of the box scheme are advantageous for reactive flow problems. These systems have the general form of conservation laws (representing the groundwater transport) with source terms (representing the chemical reactions). In typical problems the reaction rates vary widely, and at least some will be on a much smaller time scale than that implied by the advection velocities. The result is the phenomenon of reduced (or retarded) speed whereby the transport of pollutants is much slower than the flow of the groundwater. In the design of numerical methods we wish to take time-steps guided by this reduced speed. However, the standard practice of using operator splitting methods with explicit time-stepping, to maintain stability, requires much smaller time-steps than might be expected from a knowledge of the retardation behavior.

We have applied a combination of the box scheme and trapezoidal scheme (box-trap scheme) in one and two space dimensions, to both simple problems and more complex systems. The box scheme is commonly used by hydraulics engineers in the study of river modeling where the flood wave flows at a slower speed than the fluid velocity. It is unconditionally stable which allows us to choose the most appropriate time-step to best represent the speed and accuracy of the solution. Moreover, the scheme robustly handles the different speeds that can occur in larger systems. The main disadvantage of the scheme is the presence of spurious oscillations in the numerical solution. It is well-known that these oscillations can be damped by introducing a theta-weighting of the spatial derivatives and this does not noticeably affect the accuracy.

A modified equation analysis gives considerable insight into both the numerical scheme and original model itself. The resulting analysis enables us to understand and hence control and damp the oscillations. A variation of the usual modified equation analysis using the oscillatory part of the solution means we can predict the exact position of the oscillations. We have a paper accepted in SIAM Journal on Numerical Analysis and a pre-print is also available at www.iam.ubc.ca/∼sarah.

We are currently working on applying the box-trap scheme to other reaction-transport systems which also exhibit a reduced speed for certain parameter regimes. By choosing the time-step to match the reduced speed we aim to improve on existing numerical schemes, in terms of stability, accuracy and computation time. We hope that these will be of interest to the water resources research community as our suggested scheme only requires a very small modification to splitting methods which are currently used in the field. Numerical code already in existence could easily be adapted to solve the full model using the box scheme which would demand no extra computational complexity.