

CAN YOU WIN AT TETRIS?

By

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We accept this thesis as conforming
to the required standard

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Abstract

TETRIS is a popular video game in which you try to fill rows in a rectangular well using a sequence of tetrominoes chosen by the machine. Each time you succeed in filling a row, it is deleted from the well. Your game ends when you have stacked pieces up to the top of the well. I build a model of TETRIS and analyze the worst-case scenario, in which the machine is treated as an adversary. I say you have a winning strategy when you can make your game last indefinitely. I construct winning strategies for some subsets of the TETRIS pieces, and prove that none exists for some others. Finally, I compare these analytic results to some empirical average-case data that I obtain from a passive survey of TETRIS players.

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... instead of a classical concerto, I chose one of my own. While I might not be able to compete successfully in performance of a classical concerto, there was a chance that my own might impress the examiners by the novelty of technique; they simply would not be able to judge whether I was playing it well or not! On the other hand, even if I did not win, the defeat would be less mortifying since no one would know whether I had lost because the concerto was bad or because my performance was faulty. [1]