Chapter 2

Strategies

Let me introduce you to the main concept of this thesis, but first, some motivation.

2.1 Doing well at tetris and TETRIS

Your high score is one measure of your TETRIS-playing ability, but perhaps you're so talented that you can actually **beat** the game: you can make your score as high as you like. But is this really possible? Can TETRIS be beaten? This is *almost* the question I'll answer, but first, a matter of equity.

With all due respect, is it fair that you must compete against a machine? Your reflexes are no doubt spectacular, but can they hold out indefinitely against an unwavering silicon opponent when you are forced to play faster and faster using a sluggish joystick? Surely not. For there to be any hope of beating TETRIS, its pieces cannot be allowed to fall so quickly that you lose track of the game: there must be a speed limit. But since a reasonable speed limit for you might be still too fast for other players, I'm going to set it *so* low that, effectively, all players can take whatever time they need in positioning a piece. That's what I've done in tetris: the speed and timing factors of TETRIS have been eliminated. So, the fairer question I ask is "Can *tetris* be beaten?"

"Wait a minute", you implore, "how can I beat tetris when I don't even get any points?" The answer is to use the length of your game, rather than a score, as the measure of your ability: if the game ends, you have lost, otherwise, you have beaten tetris. This might seem strange, but think about what you need to beat TETRIS: making your score very high requires playing for a long time. If you can score as high as you want, you really must be able to make your game last indefinitely. Therefore, if you can beat TETRIS, you can also beat tetris. Conversely, suppose you can beat tetris. Each piece adds four new full cells to the well. If you don't keep clearing rows, then eventually, some of these full cells will be above row 20, and your game will end. Therefore, as long as you continue the game, you must clear, on average, four rows every ten plays, so that the number of full cells doesn't increase past a certain point. But if you can do this in tetris, then you can do it (with good reflexes) in TETRIS.

I could end here simply by stating "Yes, *I* (or someone else) can beat tetris", but you'd probably be less than convinced (especially if you had seen me play). Alternatively, I could state "*No one* can beat tetris", but you would doubt this even more: had I actually investigated every claim to the contrary ever made, and found each to be false? Surely what you demand is either a way of beating tetris, or a proof that it can't be done.

2.2 Telling you how to play

If there's a way to beat tetris, then I can describe it in pictures. Figure 2.4 shows what such pictures are composed of, namely state diagrams with arrows between them. In a state diagram, the well is a simple outline which I draw only as tall as I need to show you all the state's (shaded) full cells. If I need to refer to a state, I will label it with a letter just below its diagram. Between some states I draw arrows to represent a play that takes the well from the state at the arrow's tail to the one at its head. Each arrow is labelled with pieces: the current piece is the large one with a number below it, and any small ones are lookahead pieces. The best way to understand how to interpret these labels is to study Figure 2.4.

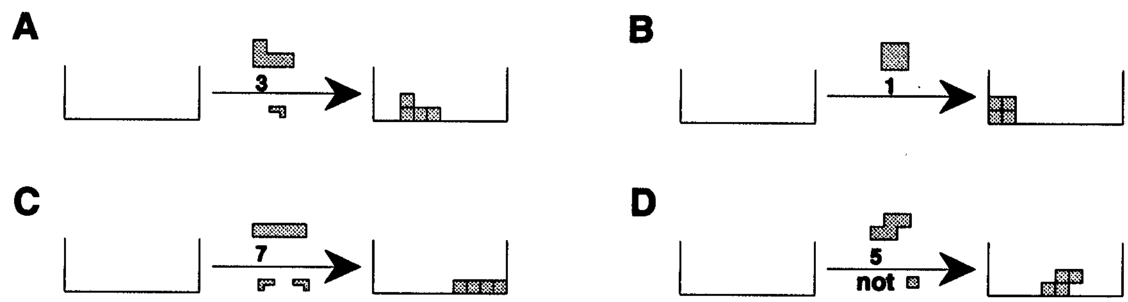

Figure 2.4: Four examples of arrows, and what their labels tell you to do. **A:** When the (big) current and (small) lookahead pieces are both right elbows, rotate the elbow into the same orientation as the big piece, and place it so that its leftmost cell goes in column 3. **B:** when the current piece is a square, and the lookahead piece is anything, play the square so that its leftmost cell goes in column 1. **C:** when the current piece is a bar and the lookahead piece is one of the elbows, play the bar horizontally, with its leftmost cell in column 7. **D:** when the current piece is a left kink and the lookahead piece is any piece other than a square, play the kink horizontally, starting in column 5.

2.3 Strategies for tetris

The pictures in Figure 2.4 are only fragments of instructions: if you try to use them, you will get stuck in the first state unless the machine gives you the same current and lookahead pieces as appear on the arrow. Even if it does, you'll be stuck after the first play, since there are no arrows leading from the second state. You can see that for a complete set of instructions, every state must have enough links leading from it that you won't get stuck, no matter what pieces the machine gives you. Such a complete set of states and links, whether or not it actually beats tetris, is what I call a **strategy**. My quest is to seek a **winning** strategy for tetris.

Any strategy must be able to deal with whatever sequence of pieces the machine sends you. I'll test my strategy building and your arrow interpretation by considering the simplest sequences, those in which all pieces are the same. Figure 2.5 shows a simple way of playing such a sequence of squares. Notice that the highest row in which a full cell is found is 2. I'll call this number the **height** of the strategy. If the square were the only piece in tetris, this would be a winning strategy, since its never fills a cell higher than row 2, let alone row 20.

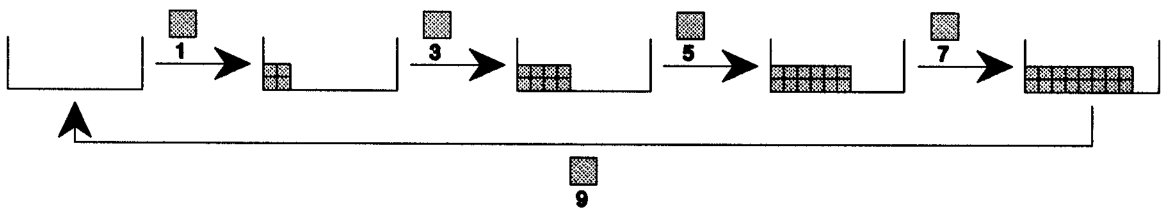

Figure 2.5: A strategy for playing only squares. You clear two rows when you play the square in column 9, and this leads you back to the empty well.

The square is not alone: if the machine sends you a sequence consisting entirely of any one of the pieces, you can make your game last indefinitely by using one of the single-piece strategies shown in Figures 2.6 to 2.9. For kinks and elbows, I've only shown a strategy for the left version of the piece. You can obtain a strategy for the right version by reversing the picture with a mirror, left to right. It's important that these strategies exist, for otherwise, the machine could quickly end your game by simply handing you the same piece over and over again.

2.4 Meaningless numbers

The size of the well in arcade TETRIS happens to be 10 by 20, but there's no obvious reason why it couldn't be different. In some versions of TETRIS, the well is wider, and so I might as well try to find winning strategies for all widths. Similarly, there seems to be nothing magical about 20: other well depths are possible (and likely). I will note

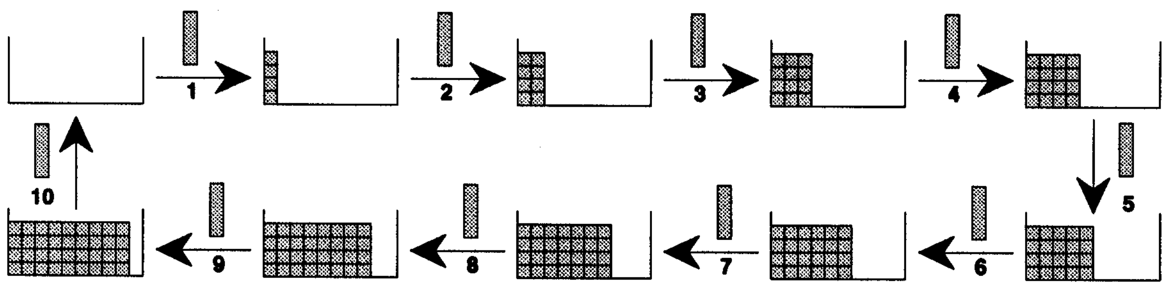

Figure 2.6: A strategy for bars with a height of 4 and 10 states.

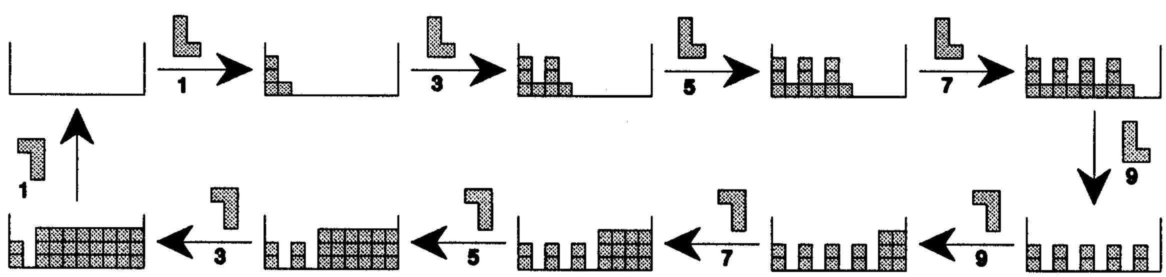

Figure 2.7: A strategy for left elbows with a height of 3 and 10 states.

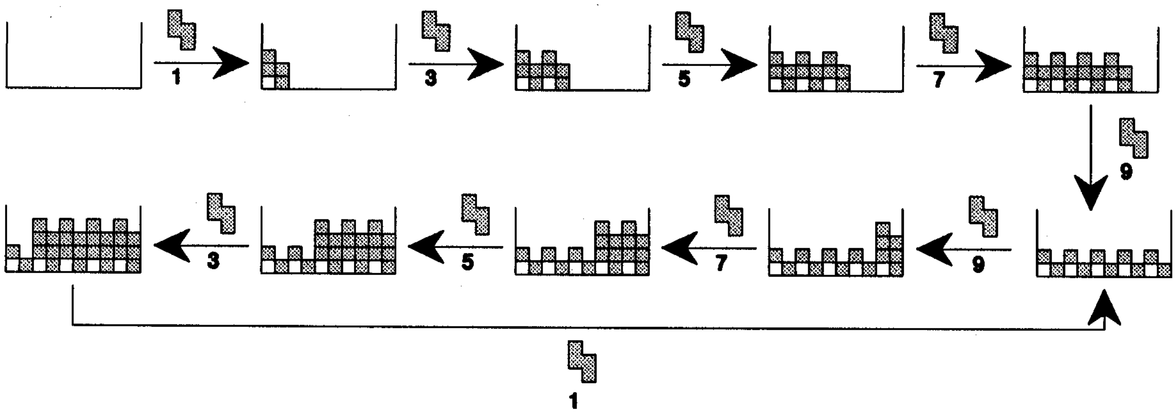

Figure 2.8: A strategy for left kinks with a height of 4 and 10 states.

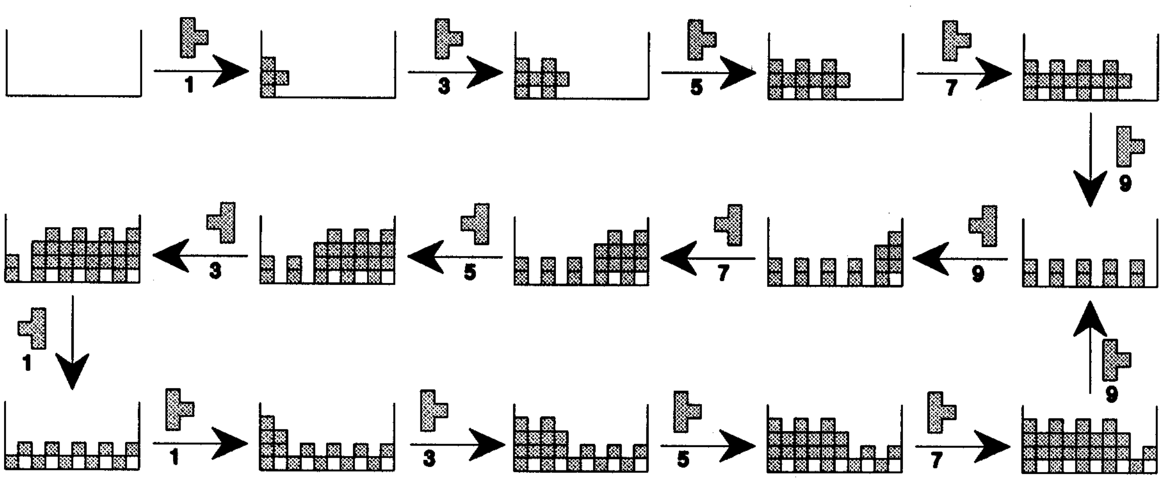

Figure 2.9: A strategy for tees with a height of 4 and 15 states.

the height of each strategy I find so that you will know the depth of well you require in order to use it. (It might be the case that I can find a winning strategy for seven-piece tetris in a well of depth 21, but that none exists for the real well of depth 20.) For single pieces, you'll see that the width of the well usually doesn't matter.

I begin by showing you that the single-piece strategies of Figures 2.5 to 2.9 easily lead to strategies for all wells of even width. You can think of the well as being partitioned into five pairs of adjacent columns which I'll call **lanes**, and will number from left to right, starting at 1. Thus lane 1 consists of columns 1 and 2, lane 2 consists of columns 3 and 4, and so on. Notice that in each strategy, pieces are placed entirely within lanes: no piece adds full cells to two different lanes. Moreover, in each strategy, the pattern of playing pieces is identical for every lane. This pattern can be repeated for any number of lanes, giving strategies for wells with any even number of columns. The height of these strategies doesn't depend on the width of the well, except for the simple case of width 2. There, the square has a zero-height strategy (it disappears upon being played), the bar has a height four strategy, and the other pieces all have strategies of height two. (Also, you might have noticed that there is a zero-height strategy for the bar in a well of width 4, and a height one strategy for the bar in wells of widths 8, 12, 16, and so on.)

What about wells of odd width? Squares immediately present a problem, since they are two cells wide, and so you can't even fill a single row with them. This settles the question: there can be no winning strategy for tetris in an odd width well, since the machine can simply send you a sequence of squares, and you'll never clear a row. (Perhaps that's why every version of TETRIS that I've ever seen has a well of even width.)

Are squares the only pieces which prevent you from winning in an odd well? The pattern of Figure 2.6 gives a strategy for bars in odd wells: just play each one vertically. Elbows and tees are more interesting, and in Figures 2.10 and 2.11, I've shown strategies for the left elbow and for the tee in wells of width 3 and 5 (as before, you get a strategy for right elbows by reflecting the left elbow strategy in a mirror). The figure captions tell you how to construct strategies for these pieces in wells of any other odd width. On the other hand, Figure 2.12 explains why there is no winning strategy for either of the kinks in a well of odd width.

The winning strategies I've found for single pieces are summarized in Table 2.1. They let you beat tetris in wells of various widths only if the machine sends just one type of piece and the well is at least as deep as the height of the strategy.

Table 2.1: A summary of winning single-piece strategies.

	Well Width=2		Well Width=$2n, n > 1$		Well Width=$2n+3, n \geq 0$	
Piece	Height	No. of States	Height	No. of States	Height	No. of States
Square	0	1	2	n	(no winning strategy)	
Kink	2	2	4	$2n$	(no winning strategy)	
Bar	4	2	4	$2n$	4	$2n+3$
Elbow	2	2	3	$2n$	8	$4n+6$
Tee	2	3	4	$3n$	7	$5n+5$

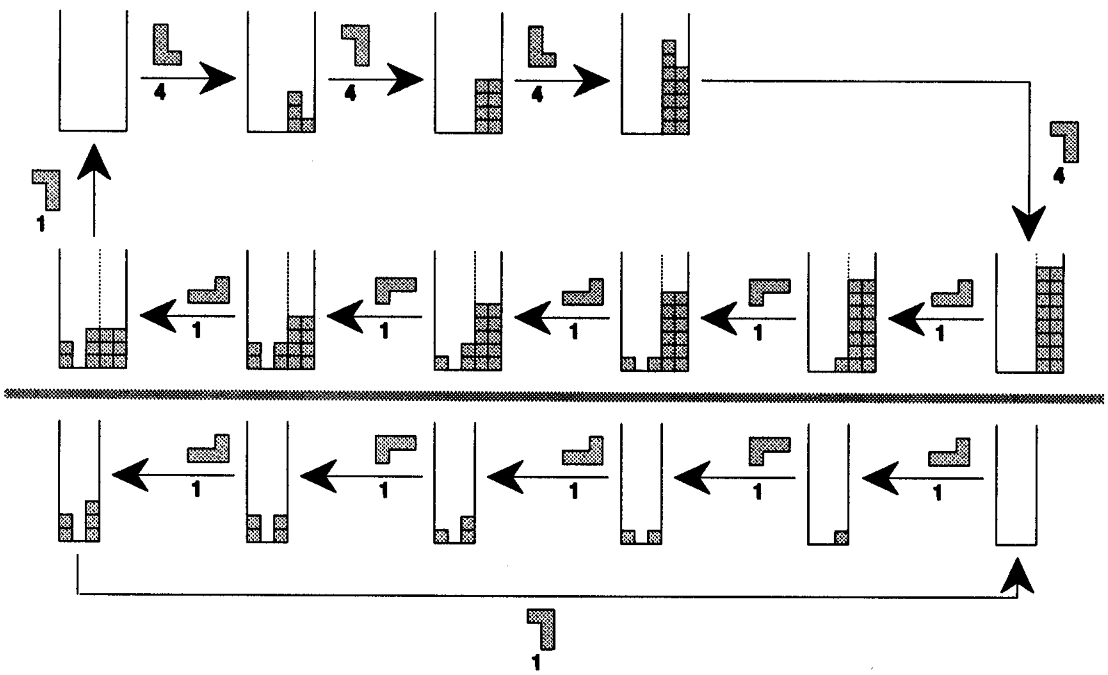

Figure 2.10: Strategies for left elbows in wells of widths 3 (below the heavy gray line) and 5 (above it). The first four plays in the width 5 strategy fill in the two rightmost columns of the well. This leaves three empty columns which can be treated just like a well of width 3. In fact, the next six plays in the width 5 strategy are exactly the same as the plays in the width 3 strategy, and to emphasize this, I've drawn them on top of each another, separated by the gray line (the dashed vertical lines inside the states of width 5 are to help you see this correspondence). For wells of greater odd width, you perform the same trick: fill pairs of columns just like you filled in columns 4 and 5 here, then use the width 3 strategy in the remaining three empty columns.

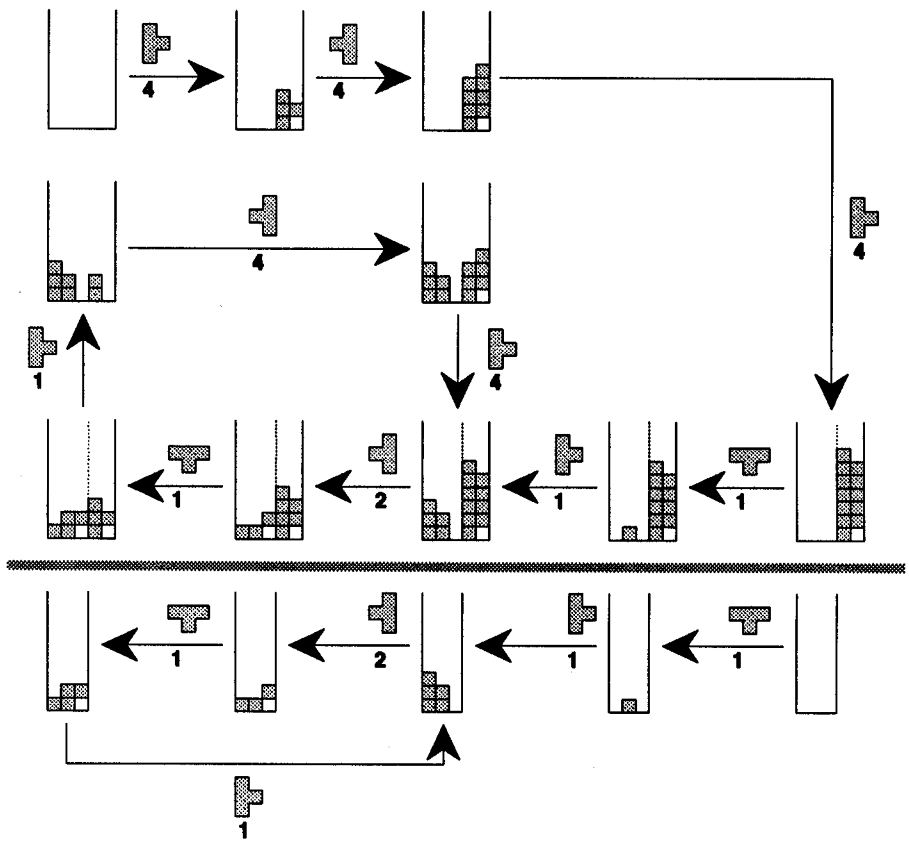

Figure 2.11: Strategies for tees in wells of widths 3 (below the heavy gray line) and 5 (above it). Just as for left elbows, the first part of the width 5 strategy serves to fill (almost) the rightmost two columns, leaving three empty columns which can be treated (almost) like a well of width 3. After the first three plays of the width 5 strategy, the next five mimic those in the width 3 strategy, and I've drawn them above each other, separated by the gray line. To construct strategies for wells of greater odd width, repeat the first three plays of the width 5 strategy in every additional pair of columns. You will then be left with three empty columns in which you can perform the width 3 strategy for the next five plays. Then, you must refill columns 4 and 5, 6 and 7, and so on, just as in the width 5 strategy.

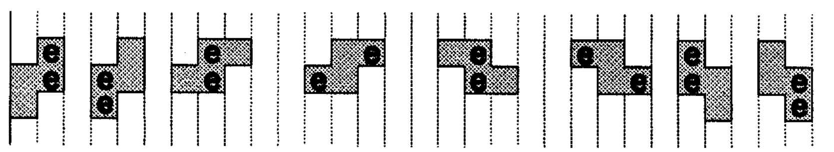

Figure 2.12: The proof that no winning strategy exists for a kink in wells of odd width is easy. Notice that no matter how you play either kink, you create the same number of <u>full cells in even columns</u> ("even cells" – labelled **e**) as in odd columns ("odd cells" – shaded but unlabelled). I call the number of even cells minus the number of odd cells the **skew**. The picture shows that when you play a kink, you don't change the skew. A full row has one more odd cell than even cell, so when you clear a row (i.e. when you remove a row of full cells from the well), you increase the skew by one. As you make more plays, the only way to prevent the skew from increasing is to stop clearing rows, but this will soon end your game. If, on the other hand, you keep increasing the skew, then the number of even cells keeps increasing too, and that will eventually end your game — either way, you lose.