

Chapter 1

Introduction

If I tell you I'm doing "applied math", you might ask, with raised eyebrows, "applied to what?". If I respond with "plants", "people", or the name of any other complicated part of the real world, your interest might well turn into suspicion. To avoid this, I've chosen to study a simple object, one whose reflection in the Platonic world of mathematical existence is clear enough that you will find no cause for alarm in the analysis I perform on it.

1.1 The Real Game

TETRIS is a video game invented by Russian mathematician Alexey Pazhitnov and first programmed by Vadim Gerasimov. I'll describe first the arcade version (©1988 by Atari Corporation, TM and ©1987 by AcademySoft-Elorg) and then the generic version which I actually analyze. Figure 1.1 shows what you'll see if you venture into an arcade for a game. After absolving yourself of a quarter and selecting single player mode, you must choose whether to begin at the **Easy**, **Medium**, or **Hard** level.

Figure 1.2 is a sample of what appears on the screen in a typical TETRIS game. The large rectangular region is the **well**, and the shaded squares in it are coloured **tiles**. (On Easy level, you begin the game with an empty well.) The well has space for 20 rows of ten tiles each. At the top of the well, you can see the **current piece**, which is made up of four tiles. The **lookahead piece** is shown just outside the well. If you don't touch the controls, the current piece will drop straight down the well until it hits a tile, or the

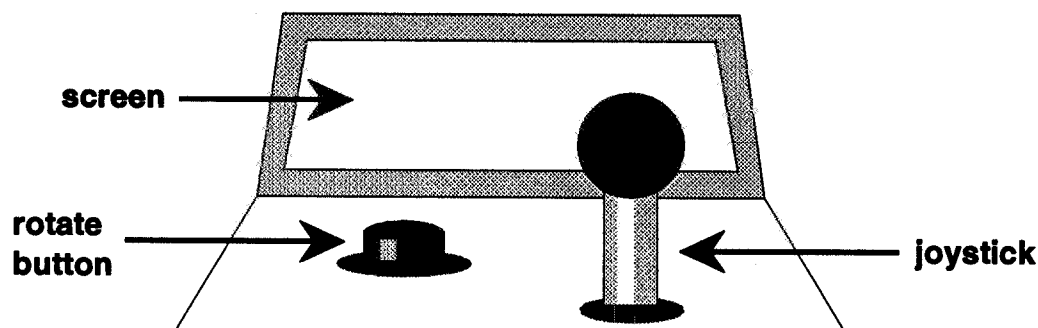


Figure 1.1: A schematic diagram of the TETRIS machine. The quarter slot is not shown.

bottom of the well, and stops. The lookahead piece then becomes the current piece at the top of the well, and a new lookahead piece is displayed outside. The game ends when tiles have accumulated to such a height that the machine can't place a new piece at the top of the well.

Most TETRIS players are not content merely to watch this (expensive) stacking, and instead, grab the controls. Hitting the **rotate button** rotates the current piece 90° clockwise as it falls (see Figure 1.3). Pushing the joystick left or right moves the current piece in that direction, again while it falls. (Pulling the joystick toward you causes the piece to fall more rapidly, and is a sign of impatience.) The point of all this is to place pieces so as to create full rows of tiles. After a piece stops falling, and before the next piece enters play, TETRIS deletes every full row of tiles from the well (you are said to have **cleared** these rows), and each tile in the well drops down one position for every row deleted below it. Clearing rows delays the end of your game, since it moves tiles away from the top of the well. Moreover, depending on how many rows (1, 2, 3, or 4) you clear with a piece, points (50, 150, 450, or 900) are added to your **score**, which is shown to the right of the well in Figure 1.2.

A game of TETRIS is divided into rounds on the basis of how many rows you've

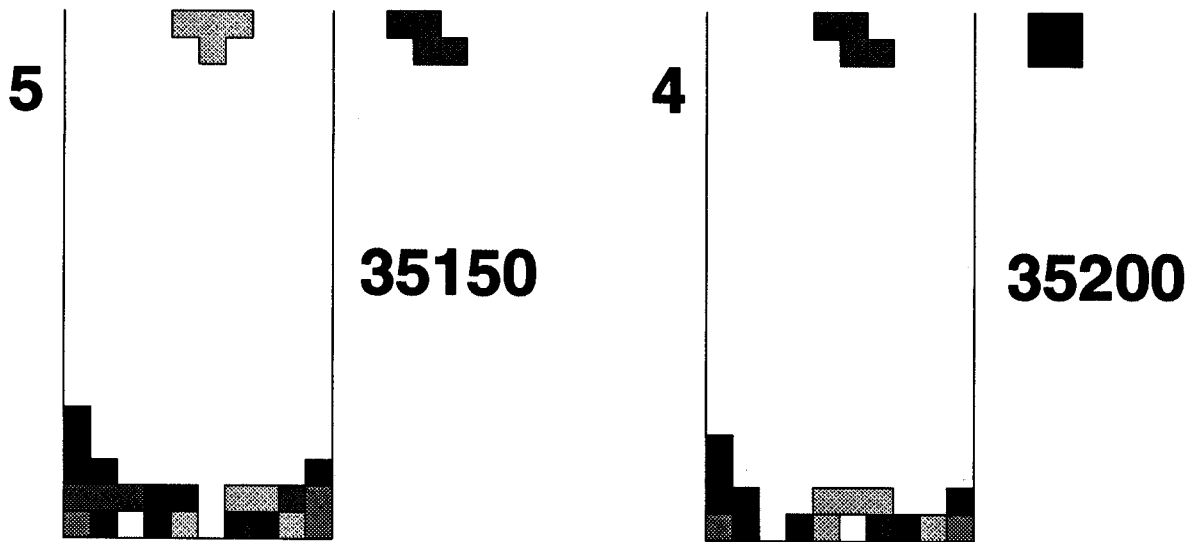


Figure 1.2: The typical TETRIS screen on the left can lead to the one on the right if you don't touch the controls. The tee has fallen straight down, filling and clearing the second row from the bottom. As a result, you are awarded 50 points, and have one less row left to clear.

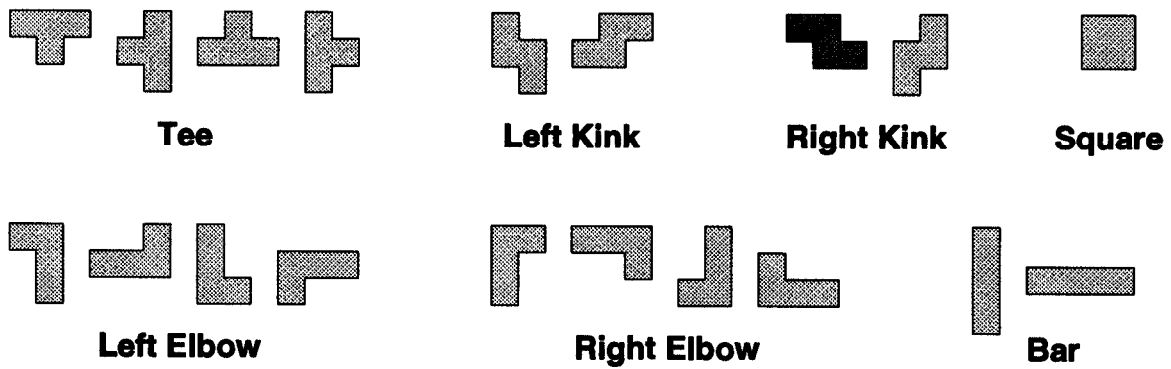


Figure 1.3: The seven TETRIS pieces are shown in all of their orientations. Hitting the rotate button gives the next orientation (in cyclic order from left to right) of a given piece.

cleared. The number still to be cleared in the current round is displayed outside the well (it's 5 in Figure 1.2). At the end of each round, you're awarded a number of bonus points that depends on the maximum height of tiles in the well at that time (the lower the tiles, the more points earned). The next round begins with a new well (which may already contain some tiles) and a new goal of rows to clear. The speed at which pieces fall generally increases from one round to the next, leaving you less and less time to decide where to place them. Moreover, some rounds include extra complications: the machine adds tiles to the well at random locations and times, or it inserts extra rows of tiles at the bottom of the well, pushing up any tiles already in the well.

In most implementations of TETRIS for microcomputers and home video game units, these complications don't exist. Rather, the entire game is just like the first round of the Easy level of arcade TETRIS: you begin with an empty well, and the game ends when you have filled cells up to its top. It's this **generic** version that I'll call "TETRIS" from now on.

1.2 TETRIS meets Plato

In constructing a mathematical model of the game, I'll make several simplifications and idealizations. You should recognize TETRIS behind this axiomatic façade, but I postpone a full discussion of its validity until Chapter 6, where the opinions of other TETRIS players are included. My abstraction of TETRIS, namely **tetris**, follows.

First, the **well** is a rectangular array of **cells**, each of which is either **full** or **empty** (so I've removed colour). **Rows** of cells are numbered from the bottom of the well, and **columns** from its left, both starting at 1. The well has no fixed depth, so the rows of empty cells extend to the heavens, but pieces enter into play just above the highest full cells. The pattern of empty and full cells in the well is called its **state**, and changes as

the game proceeds.

Next, the rules of movement are just as in TETRIS: you can move a piece down, left, and right, as well as rotate it in 90° increments, until you decide to let it drop. (Disputes over the validity of rotations in the vicinity of full cells can be resolved by a Euclidean referee: the well and the piece are copied to the plane, and a rotation in the original well is allowed only if it can be performed in the plane without ever having the piece intersect a full cell.) The piece then falls straight down, without further rotation, until its descent is stopped by either a full cell or the well floor. The status of the four cells it occupies then changes from empty to full, and if this fills any rows, they are deleted and the well compressed, just as in TETRIS. I let you move pieces left and right as far as you wish, but notice that gravity still acts once you drop them. More importantly, I've eliminated time as a factor: you can take as long as you want to play a piece.

Finally, the game ends when you fill a cell above row 20; this is essentially the same as in TETRIS. To summarize, a game of tetris is a sequence of **plays**, each of which proceeds as follows:

- the machine hands you a piece, while displaying the next piece you'll be given
- you put the piece into the well above all full cells, then slide and rotate it into any position, never moving it through a full cell
- you let go of the piece, and it falls straight down as far as possible
- the machine deletes each full row, moving all rows above it down one position
- if there is a full cell above row 20, the game ends

You'll notice that I've said nothing about tetris scores. This is because there won't be any! The reasons why not lead to the central concept of my thesis, and you'll find them in the next chapter.