DYNAMICS AND CONTROL OF A FLEXIBLE TETHERED SYSTEM WITH OFFSET

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A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE

in

THE FACULTY OF GRADUATE STUDIES INSTITUTE OF APPLIED MATHEMATICS

We accept this thesis as conforming to the required standard

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THE UNIVERSITY OF BRITISH COLUMBIA

August 1991

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Abstract

A mathematical model of a platform based flexible tethered satellite system in an arbitrary orbit, undergoing planar motion, is obtained using the Lagrangian procedure. The governing equations of motion account for the platform and tether pitch, longitudinal tether oscillations, offset of the tether attachment point as well as deployment and retrieval of the tether.

A numerical parametric study of the highly nonlinear, nonautonomous and coupled equations of motion gives considerable insight into the system dynamics useful in its design. Of particular interest are the interactions involving orbital eccentricity, system librations, tether flexibility and offset, retrieval maneuvers and initial disturbances. Results show that the offset strongly couples tether and platform dynamics, and the resulting responses show high frequency modulations corresponding to the longitudinal tether oscillations. The system was found to be unstable during retrieval. The Linear Quadratic Regulator based offset control strategy, in conjunction with the platform mounted momentum gyros, is proposed to alleviate the situation. Results show that a strategy involving independent parallel control of low and high frequency responses can damp rather severe disturbances in a fraction of an orbit.
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List of Symbols

[A] \text{coefficient matrix of } x \text{ and } x̄

a_p \text{ altitude at perigee}

α_p \text{ platform pitch angle}

α_t \text{ tether pitch angle}

[B], [B̅] \text{coefficient matrix of } u \text{ and } ū

C \text{ system center of mass}

[C] \text{coefficient matrix of q}

d_x, d_z \text{ horizontal and vertical offsets, respectively}

D_x, D_z \text{nondimensionalized offsets; } d_j/l_b, j = x, z

d \text{ vector of offsets, } d = d_x i_p + d_z k_p

e \text{ orbit eccentricity}

ε \text{ tether strain variable}

G \text{ universal gravitational constant}

h_K \text{ orbit constant}

i_j, j_j, k_j \text{ unit vectors in frame } F_j, j = i, c, p, t

I_{xx}, I_{yy}, I_{zz} \text{ platform inertias}

[I], [0] \text{identity and zero matrices, respectively}

[K] \text{coefficient matrix of q, stiffness matrix}

l, l \text{ instantaneous tether line vector and magnitude, respectively}

\bar{l} \text{ nominal unstretched tether length}

L, \bar{L} \text{nondimensional forms of } l \text{ and } \bar{l}; \ L = l/l_b, \ \bar{L} = l/\bar{l}_b

l_b \text{ initial nominal tether length}

M_e \text{ mass of earth}
\([M]\) \hspace{1cm} \text{coefficient matrix of } \dot{\bar{q}}, \text{ mass matrix}

\(M_s\) \hspace{1cm} \text{subsatellite mass}

\(M_r\) \hspace{1cm} \text{reel mass}

\(M_t\) \hspace{1cm} \text{deployed tether mass}

\(M_p\) \hspace{1cm} \text{platform mass}

\(M_{srt}\) \hspace{1cm} \(M_s + M_r + M_t\)

\(M_{pct}\) \hspace{1cm} \(M_p + M_r + M_t\)

\(M\) \hspace{1cm} \text{total mass of the system, } M = M_p + M_r + M_t + M_s

\(\bar{P}, P\) \hspace{1cm} \text{retrieval and eccentricity influence vectors, respectively}

\(q\) \hspace{1cm} \text{vector of generalized coordinates}

\([Q]\) \hspace{1cm} \text{matrix of weights for control variables}

\([R]\) \hspace{1cm} \text{matrix of weights for state variables}

\(\rho\) \hspace{1cm} \text{tether line density}

\(\tau\) \hspace{1cm} \text{nondimensional platform wheel torque}

\(T\) \hspace{1cm} \text{system kinetic energy}

\(U_g, U_s\) \hspace{1cm} \text{gravitational and strain energies, respectively}

\(U\) \hspace{1cm} \text{system potential energy, } U = U_g + U_s

\(u\) \hspace{1cm} \text{vector of control variables}

\(\omega_t\) \hspace{1cm} \text{angular velocity of the tether frame}
Acknowledgement

I would like to offer my sincere thanks to Dr. Vinod Modi for his direction in the preparation of this thesis. I would also like to express my appreciation to the Institute of Applied Mathematics for providing an environment which encourages work in fields of application. Finally a special thanks to Toni Foster for her invaluable support and encouragement.