Chapter 3

Parametric Study

3.1 Introduction

Studying the uncontrolled dynamics of the system is important for several reasons. First, it provides a better understanding of the system. For example, the existence and strength of coupling and the presence of resonance due to similar frequencies in the system can be discovered. Secondly, the dynamics will reveal if there is a need for control of the system and may suggest what type would be the best. Finally, if control is necessary, the uncontrolled dynamics provide a comparison as to how well the control strategies are working. The study is initiated with the system in stationkeeping mode. That is the tether is neither being deployed nor retrieved. Some important initial system parameters are listed in the following table.

Table 3.1 System characteristics.

<table>
<thead>
<tr>
<th>Platform and Subsatellite Characteristics</th>
<th>Tether Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Platform Mass: 100,000 kg</td>
<td>Status: Station Keeping</td>
</tr>
<tr>
<td>Platform Inertias:</td>
<td>Diameter: 0.002 m</td>
</tr>
<tr>
<td>$I_{xx} = 1.2 \times 10^8 \text{kgm}^2$</td>
<td>Young's Modulus: 1.25\times10^8 \text{Nm}</td>
</tr>
<tr>
<td>$I_{yy} = 2.0 \times 10^8 \text{kgm}^2$</td>
<td>Linear Density: 2 \text{kg/km}</td>
</tr>
<tr>
<td>$I_{zz} = 8.3 \times 10^7 \text{kgm}^2$</td>
<td>Equilibrium Length: 1 \text{ km}</td>
</tr>
<tr>
<td>Satellite Mass: 100 kg</td>
<td></td>
</tr>
</tbody>
</table>

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It should be noted here that the tether oscillations are at a higher frequency than the pitch oscillations. For the 1 km tether modeled here, the frequency is about 100 cycles per orbit, while the pitch motions have a frequency of about 1 cycle per orbit. It is thus difficult to resolve both of the motions on the same chart. If the high frequency response is of interest in a particular case, a second graph with a larger scale is included.

3.2 Basic Response

Figure 3.1 shows the response of the system to an initial disturbance in each of the degrees of freedom. Figure 3.1(a) shows periodic oscillations in the pitch motions as the gravity gradient torque attempts to bring the system back to the equilibrium configuration. The oscillations have constant amplitude since the inherent damping of the system was purposely ignored to accentuate the response. If necessary of course, the energy dissipation can easily be modeled through the corresponding generalized force. Figure 3.1(b) demonstrates the decoupling of platform and tether dynamics for the case of zero offsets. As expected the tether oscillations have no effect on the platform pitch.

3.3 Offsets

3.3.1 Horizontal Offset

Figure 3.2 shows the system response with the tether attachment point displaced 20 meters along the local horizontal. Notice that the platform no longer oscillates about zero(Figure 3.2(a)). The offset causes the system to rotate to a new equilibrium configuration. Coupling between the platform and longitudinal tether dynamics results in small modulations of the platform response at the tether frequency(Figure 3.2(b)). Even with a small subsatellite mass which is a tiny fraction of the platform mass, the pitch response of the platform is modulated.
<table>
<thead>
<tr>
<th>OFFSETS</th>
<th>MASS PARAMETERS</th>
<th>INITIAL CONDITIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_x = 0 \text{ M}$</td>
<td>$M_p = 100,000 \text{ KG}$</td>
<td>$\alpha_p(0) = 1^\circ$</td>
</tr>
<tr>
<td>$D_z = 0 \text{ M}$</td>
<td>$M_s = 100 \text{ KG}$</td>
<td>$\alpha_t(0) = 10^\circ$</td>
</tr>
<tr>
<td></td>
<td>$M_r = 50 \text{ KG}$</td>
<td>$\epsilon(0) = .01$</td>
</tr>
<tr>
<td></td>
<td>$\rho = .002 \text{ KG/M}$</td>
<td>$l_b = 1000 \text{ M}$</td>
</tr>
</tbody>
</table>

**ORBIT PARAMETERS**

- $e = 0$
- $h = 500 \text{ KM}$

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**Figure 3.1:** Response of the system during the reference stationkeeping configuration to a prescribed disturbance: (a) low frequency platform and tether pitch oscillations.
<table>
<thead>
<tr>
<th>OFFSETS</th>
<th>MASS PARAMETERS</th>
<th>INITIAL CONDITIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_x = 0$ M</td>
<td>$M_p = 100,000$ KG</td>
<td>$\alpha_p(0) = 1^\circ$</td>
</tr>
<tr>
<td>$D_z = 0$ M</td>
<td>$M_s = 100$ KG</td>
<td>$\alpha_t(0) = 10^\circ$</td>
</tr>
<tr>
<td>ORBIT PARAMETERS</td>
<td>$M_r = 50$ KG</td>
<td>$\epsilon(0) = 0.01$</td>
</tr>
<tr>
<td>$e = 0$</td>
<td>$\rho = 0.002$ KG/M</td>
<td>$l_b = 1000$ M</td>
</tr>
<tr>
<td>$h = 500$ KM</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 3.1: Response of the system during the reference stationkeeping configuration to a prescribed disturbance: (b) relatively high frequency longitudinal oscillations of the tether.
Figure 3.2: Effect of the tether attachment point’s offset along the local horizontal on the system response: (a) time history of the pitch motion.
Figure 3.2: Effect of the tether attachment point's offset along the local horizontal on the system response: (b) coupling between the tether longitudinal dynamics and the pitch motions.
3.3.2 Vertical Offset

Figure 3.3 shows the response for a vertical offset (i.e. offset along the local vertical) of 20 meters. The equilibrium position of the platform remains unaffected in this case. Note the effect of the tether stretch on the platform is now less pronounced than that for the horizontal offset case. This can be expected as the torque applied to the platform is primarily governed by the offset along the local horizontal.

3.4 Eccentricity

Eccentricity has the effect of introducing a periodic (at the orbital frequency) forcing term into the pitch equations. To study the effect of eccentric orbits the offsets and initial disturbances are set to zero. Figure 3.4 compares the response for orbits with $e = 0.01$ and $e = 0.05$. As anticipated, the higher eccentricity increases the amplitude of the response, particularly in the platform pitch. For $e = 0.05$, $\alpha_p$ reaches $30^\circ$ which may not be acceptable. However, the tether pitch response is confined to $4^\circ$ even for $e = 0.05$. As expected, the tether’s longitudinal mode remained virtually unexcited due to the eccentricity.

3.5 Subsatellite Mass

Figure 3.5 shows the effect of doubling the subsatellite mass. Note, the platform pitch angle reaches a much lower value for $M_s = 200$ kg. This is to be expected since the extra mass increases the restoring gravity gradient moment. With the horizontal offset and the subsatellite mass, the platform equilibrium position is also affected. The period of the longitudinal tether oscillations increases by about 30 percent for $M_s = 200$ kg (Figure 3.5(b)). The increased mass also causes the high frequency platform pitch modulation to be a little more pronounced.
<table>
<thead>
<tr>
<th>OFFSETS</th>
<th>MASS PARAMETERS</th>
<th>INITIAL CONDITIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_x = 0$ M</td>
<td>$M_p = 100,000$ KG</td>
<td>$\alpha_p(0) = 1^\circ$</td>
</tr>
<tr>
<td>$D_z = 20$ M</td>
<td>$M_s = 100$ KG</td>
<td>$\alpha_i(0) = 1^\circ$</td>
</tr>
<tr>
<td></td>
<td>$M_r = 50$ KG</td>
<td>$\epsilon(0) = .01$</td>
</tr>
<tr>
<td><strong>ORBIT PARAMETERS</strong></td>
<td>$\rho = .002$ KG/M</td>
<td>$l_b = 1000$ M</td>
</tr>
</tbody>
</table>

$e = 0$
$h = 500$ KM

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![Graph](image1)

**Figure 3.3:** Effect of the tether attachment point’s offset along the local vertical on the system response: (a) time history of the pitch motion.
Figure 3.3: Effect of the tether attachment point's offset along the local vertical on the system response: (b) small influence of the tether's longitudinal dynamics on its pitch motion.
OFFSETS
$D_x = 0$ M
$D_z = 0$ M

MASS PARAMETERS
$M_p = 100,000$ KG
$M_s = 100$ KG
$M_r = 50$ KG
$\rho = .002$ KG/M

ORBIT PARAMETERS
$h = 500$ KM

INITIAL CONDITIONS
$\alpha_p(0) = 0^\circ$
$\alpha_l(0) = 0^\circ$
$\epsilon(0) = 0$
$I_b = 1000$ M

LEGEND
$e = .01$
$e = .05$

Figure 3.4: System pitch response as influenced by the orbit eccentricity.
Figure 3.5: System dynamics as affected by the subsatellite mass: (a) pitch response over a long duration.
Figure 3.5: System dynamics as affected by the subsatellite mass: (b) enlarged view over a short duration showing the coupling effects.
3.6 Tether Mass

The effect of a more massive tether on the dynamics was studied by increasing its line density from 0.002 kg/m to 0.2 kg/m (Figure 3.6). This increases the tether mass from 2 kg to 200 kg. Notice that the influence is very similar to that of increasing the subsatellite mass (Figure 3.5). In fact, many investigators approximate the effect of the tether mass simply by increasing the mass of the subsatellite. This is called the lumped mass approach. Note, however, that the high frequency platform pitch modulations are not as much effected by the change as they were in the subsatellite mass variation case. In both of the cases the tether pitch oscillations remain relatively unaffected.

3.7 Platform Inertias

The inertias used thus far are the same as in reference [6] and are intended to model a space station. Since the mass is spread out (the platform is modelled as a rectangular plate), the inertias are relatively large causing resistance to high frequency disturbances. Here the inertias are changed to approximate those of the U.S. Space Shuttle:

\[
I_{xx} = 8.5 \times 10^6 \text{kgm}^2 \\
I_{yy} = 8.5 \times 10^6 \text{kgm}^2 \\
I_{zz} = 1.1 \times 10^6 \text{kgm}^2
\]

The inertias correspond to an orientation which has the Shuttle's nose pointed directly away from the earth and the wings in the plane of the orbit (Lagrange configuration; Minimum moment of inertia along the local vertical, maximum moment of inertia along the orbit normal). This has been shown to be the most stable configuration [11]. The platform mass is also changed to match that of the shuttle. The smaller inertias mean
that the restoring moment due to the gravity gradient is smaller. For a given offset, the shuttle deviates from the reference equilibrium by a significant amount, as shown in Figure 3.7. To partially compensate for this the offsets were reduced to 10 meters in each direction. The smaller inertias also cause the frequency of the platform pitch to increase from 0.9 cycles/orbit to 1.5 cycles/orbit. Figure 3.7(b) shows how decreasing the inertias can increase the coupling between the shuttle pitch and the tether stretch. Even with smaller offsets the high frequency shuttle pitch oscillations have a much larger amplitude than those of the platform (Figure 3.5(b)).

3.8 Reel Mass

The reel mass was increased from 50 kg to 500 kg and the results plotted in Figure 3.8. Notice that the platform pitch angle is affected because of the reel location offset from the platform center of mass. The increased reel mass changes the inertias of the platform thus affecting its equilibrium orientation. Of course the tether pitch equilibrium is not affected.

3.9 Tether Length

The effect of tether length on the dynamics is studied by comparing the dynamics for \( l_0 = 1000 \) m and \( l_0 = 100 \) m (Figure 3.9). The shorter tether length results in a reduced gravity gradient torque. Also the platform equilibrium position is less affected by the tether offset (Figure 3.9(a)). Another effect of the decreased gravity gradient force is an increase in the period of the tether pitch. Figure 3.9(b) shows that the frequency of the longitudinal tether oscillations for a 100 m tether is three times that of a 1000 m tether. Note also that the amplitude of the high frequency superimposed modulations decreases as the length decreases for a given initial strain (\( \epsilon = 0.01 \)).
OFFSETS
\( D_x = 20 \text{ M} \)
\( D_z = 20 \text{ M} \)

ORBIT PARAMETERS
\( e = 0.01 \)
\( h = 500 \text{ KM} \)

MASS PARAMETERS
\( M_p = 100,000 \text{ KG} \)
\( M_s = 100 \text{ KG} \)
\( M_r = 50 \text{ KG} \)

INITIAL CONDITIONS
\( \alpha_p(0) = 2.12^\circ \)
\( \alpha_t(0) = 1^\circ \)
\( \tau(0) = 0.01 \)
\( l_b = 1000 \text{ M} \)

**Legend**
\( \rho = 0.002 \text{ KG/M} \)
\( \rho = 0.2 \text{ KG/M} \)

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**Figure 3.6:** Effect of tether mass on the system response: (a) time history of the platform and tether pitch dynamics.
<table>
<thead>
<tr>
<th>OFFSETS</th>
<th>MASS PARAMETERS</th>
<th>INITIAL CONDITIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_x = 20$ M</td>
<td>$M_p = 100,000$ KG</td>
<td>$\alpha_p(0) = -2.12^\circ$</td>
</tr>
<tr>
<td>$D_z = 20$ M</td>
<td>$M_s = 100$ KG</td>
<td>$\alpha_l(0) = 1^\circ$</td>
</tr>
<tr>
<td></td>
<td>$M_r = 50$ KG</td>
<td>$\epsilon(0) = .01$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$l_b = 1000$ M</td>
</tr>
</tbody>
</table>

**ORBIT PARAMETERS**

| $e = .01$ |
| $h = 500$ KM |

**LEGEND**

$\rho = .002$ KG/M  
$\rho = .2$ KG/M

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Figure 3.6: Effect of tether mass on the system response: (b) longitudinal dynamics of the tether and its coupling effects.
<table>
<thead>
<tr>
<th>OFFSETS</th>
<th>MASS PARAMETERS</th>
<th>INITIAL CONDITIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>(D_x = 10 \text{ M})</td>
<td>(M_s = 100 \text{ KG})</td>
<td>(\alpha_t(0) = 1^\circ)</td>
</tr>
<tr>
<td>(D_z = 10 \text{ M})</td>
<td>(M_r = 50 \text{ KG})</td>
<td>(\varepsilon(0) = 0.01)</td>
</tr>
<tr>
<td></td>
<td>(\rho = 0.002 \text{ KG/M})</td>
<td>(l_b = 1000 \text{ M})</td>
</tr>
</tbody>
</table>

**ORBIT PARAMETERS**

- \(e = 0.01\)
- \(h = 500 \text{ KM}\)

**LEGEND**

- PLATFORM
- SHUTTLE

---

**Figure 3.7:** System response showing the effect of platform inertias: (a) pitch response.
OFFSETs  
$D_x = 10 \text{ M}$  
$D_z = 10 \text{ M}$  

MASS PARAMETERS  
$M_p = 79,000 \text{ KG}$  
$M_z = 100 \text{ KG}$  
$M_r = 50 \text{ KG}$  
$\rho = 0.002 \text{ KG/M}$

INITIAL CONDITIONS  
$\alpha_p(0) = -6.9^\circ$  
$\alpha_t(0) = 1^\circ$  
$\varepsilon(0) = 0.01$  
$l_b = 1000 \text{ M}$

ORBIT PARAMETERS  
e = .01  
h = 500 KM

SHUTTLE INERTIAS

Figure 3.7: System response showing the effect of platform inertias: (b) high frequency coupling effects of the tether longitudinal dynamics.
Figure 3.8: Effect of the reel mass on the system dynamics.
<table>
<thead>
<tr>
<th>OFFSETS</th>
<th>MASS PARAMETERS</th>
<th>INITIAL CONDITIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_x = 20$ M</td>
<td>$M_p = 100,000$ KG</td>
<td>$\alpha_p(0) = -2.12^\circ$</td>
</tr>
<tr>
<td>$D_z = 20$ M</td>
<td>$M_I = 50$ KG</td>
<td>$\alpha_t(0) = 1^\circ$</td>
</tr>
<tr>
<td>$M_s = 100$ KG</td>
<td>$\rho = .002$ KG/M</td>
<td>$\epsilon(0) = .01$</td>
</tr>
<tr>
<td>$e = .01$</td>
<td>$h = 500$ KM</td>
<td></td>
</tr>
</tbody>
</table>

**ORBIT PARAMETERS**

<table>
<thead>
<tr>
<th>$I_b = 1000$ M</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_b = 100$ M</td>
</tr>
</tbody>
</table>

**LEGEND**

**Figure 3.9:** Effect of the tether length on the response of the system: (a) pitch motion.
## Offsets
- \(D_x = 20\) M
- \(D_z = 20\) M

## Mass Parameters
- \(M_p = 100,000\) KG
- \(M_r = 50\) KG
- \(M_s = 100\) KG
- \(\rho = 0.002\) KG/M

## Initial Conditions
- \(\alpha_p(0) = -2.12^0\)
- \(\alpha_l(0) = 1^0\)
- \(\epsilon(0) = 0.01\)

## Orbit Parameters
- \(e = 0.01\)
- \(h = 500\) KM

### Legend
- \(l_b = 1000\) M
- \(l_b = 100\) M

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### Figure 3.9: Effect of the tether length on the response of the system: (b) coupling effects due to change in the tether longitudinal oscillation frequency.
3.10 Retrieval

Retrieval of a deployed tether is a difficult task. As the length decreases any disturbance in the tether pitch must increase in order to conserve the angular momentum of the system. In the equations of motion, the coefficient of $\alpha_t$ becomes negative during retrieval thus imparting, effectively, negative damping to this degree of freedom. Retrieval is achieved by supplying the desired nominal length function ($\bar{l}$) in the equations of motion. Decaying exponential schemes are desirable in applications since they avoid quick decelerations at the end of the maneuver. In this study, the nominal length is given by

$$\bar{l} = l_b \exp [ct],$$

where $t$ is time in seconds and $c$ is a constant (negative for retrieval, positive for deployment).

Since it is more convenient to specify the retrieval time in orbits ($t \approx \theta a_p^{1.5} (GM_e)^{-0.5}$), the above equation can be rewritten as

$$\bar{l} = l_b \exp \left[ \frac{c \theta a_p^{1.5}}{\sqrt{GM_e}} \right],$$

where $\theta$ represents the true anomaly in orbital units.

For example, if it is desired to reduce a tether's nominal length from 100 m to 10 m in 0.4 orbit, one can solve for $c$ from

$$10 = 100 \exp \left[ \frac{.4c a_p^{1.5}}{\sqrt{GM_e}} \right]$$

to give $c = -0.00638$.

In order to study the effect of the retrieval rate on the system dynamics, a small initial disturbance of $1^\circ$ is given to the tether pitch without affecting the platform. Note, in Figure 3.10, the tether pitch angle quickly reaches $13^\circ$. The platform also librates due to
coupling through the offset. The offset here is in the $z$ direction, a relatively less critical situation. As mentioned earlier, retrieval maneuvers represent a critical phase leading to instability if uncontrolled. Any practical application of tethers will have to address this problem effectively.
<table>
<thead>
<tr>
<th>OFFSETS</th>
<th>MASS PARAMETERS</th>
<th>INITIAL CONDITIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_x = 0 \text{ M}$</td>
<td>$M_p = 100,000 \text{ KG}$</td>
<td>$\alpha_p(0) = 0^\circ$</td>
</tr>
<tr>
<td>$D_z = 5 \text{ M}$</td>
<td>$M_s = 100 \text{ KG}$</td>
<td>$\alpha_t(0) = 1^\circ$</td>
</tr>
<tr>
<td>$M_r = 50 \text{ KG}$</td>
<td>$\rho = .002 \text{ KG/M}$</td>
<td>$\epsilon(0) = 0$</td>
</tr>
<tr>
<td>$e = .01$</td>
<td>$l_b = 100 \text{ M}$</td>
<td></td>
</tr>
<tr>
<td>$h = 500 \text{ KM}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Figure 3.10:** Retrieval from 100 m to 10 m in .4 orbits