Appendix B: Typical Weighting Matrices

The decomposition described in Section 4.2 amounts to writing the functional $J$ of Section 4.1 as $J = J_s + J_f$, where $J_s$ corresponds to the lower frequency pitch motions and $J_f$ pertains to the tether stretch. These two quantities are given by:

\[
J_s = \int_0^\infty x_s^T(Q_s)x_s + u_s^T R_s u_s \, dt;
\]
\[
J_f = \int_0^\infty x_f^T(Q_f)x_f + u_f^T[R_f]u_f \, dt.
\]

The nonzero elements of the weighting matrices are given here for the stationkeeping case shown in Figure 4.3 (fixed weight case). Note that $[R_f]$ and $u_f$ are scalars in this case since $D_s$ is the only control variable for the high frequency motion. The feedforward matrices used to return the offsets to the starting position are also shown.

Weights for State Variables:

\[
Q_s(1,1) = 100
\]
\[
Q_s(2,2) = 1000
\]
\[
Q_s(3,3) = 100
\]
\[
Q_s(4,4) = 1000
\]
\[
Q_f(1,1) = 100
\]
\[
Q_f(2,2) = 10
\]
Weights for Control Variables:

\[ R_s(1,1) = 5 \]
\[ R_s(2,2) = 0.1 \]
\[ R_f = 0.0001 \]

Feedforward Matrices:

\[ V(1,1) = 14 \]
\[ V(2,2) = 250 \]
\[ W(1,1) = 6 \]
\[ W(2,2) = 240 \]